Online Supplement to "Empirical evidence on the Euler equation for investment in the US"

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A The investment Euler equation

A representative household's lifetime utility, separable in consumption, C_t, and hours worked, L_t, is expressed as

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t), \tag{CS 1}$$

where β is the discount factor. The household's period t budget constraint is

$$C_t + I_t + \frac{B_{t+1}}{P_t} \leq \frac{R_{t-1} B_t}{P_t} + \frac{W_t L_t}{P_t} + \Pi_t + r_t^k u_t \hat{K}_t - a(u_t) \hat{K}_t, \tag{CS 2}$$

where I_t is investment, B_t is the amount of risk-free bonds that pay a nominal gross interest rate of R_t, W_t is the nominal wage, Π_t denotes firms' profits net of lump-sum taxes, r_t^k is the real rental rate of capital, $\hat{K}_t$ is the physical capital stock, and $a(u_t)$ is the function that measures the cost of capital utilization per unit of physical capital. Capital owning households choose the capital utilization rate, u_t, that transforms physical capital $\hat{K}_t$ into effective capital K_t as follows

$$K_t = u_t \hat{K}_t. \tag{CS 3}$$

Effective capital is rented to intermediate goods producers at the rate r_t^k. Standard assumptions are: i) $\bar{u} = 1$ and $a(\bar{u}) = 0$, where a bar over a variable denotes its steady state value; ii) the curvature of the function $a(u)$, given by $a''(u)/a'(u)$, measures the elasticity of capital utilization cost and it is such that $\zeta = a''(1)/a'(1) > 0$.

The representative household accumulates end-of-period t capital according to a standard capital accumulation equation

$$\hat{K}_{t+1} = \nu_t \left[1 - \mathcal{S}\left(\frac{I_t}{I_{t-1}}\right)\right] I_t + (1-\delta)\hat{K}_t, \tag{CS 4}$$

where δ is the depreciation rate and ν_t is the investment-specific technology shock, that is, a shock to the efficiency with which the final good can be transformed into physical capital, as in JPT. The log of the investment shock follows the autoregressive stochastic process $\log \nu_t = \rho \log \nu_{t-1} + \varepsilon_t^v$, where ρ is the autoregressive coefficient.

The IAC is specified as

$$\mathcal{S}\left(\frac{I_t}{I_{t-1}}\right) I_t = \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 I_t. \tag{CS 5}$$

where the IAC function $\mathcal{S}(\cdot)$ is such that $\mathcal{S}(1) = \mathcal{S}'(1) = 0$ with $\kappa = \mathcal{S}''(1) > 0$. Here, κ, the adjustment cost parameter, denotes the inverse of the elasticity of investment with respect to the shadow price of capital. There are no adjustment costs at the steady state when I is fixed.

The representative household chooses I_t, u_t, $\hat{K}_{t+1}$, and B_{t+1} to maximise (CS 1) under the period-by-period budget constraint (CS 2) and capital accumulation equation (CS 4). Appendix A.1 shows how log-linearizing the first-order conditions of this problem and rearranging them yields the dynamic equation for investment (2)

$$\Delta\widetilde{i}_t = (\beta + \phi_q)\, E_t \Delta\widetilde{i}_{t+1} - \beta\phi_q E_t \Delta\widetilde{i}_{t+2} + \frac{1}{\kappa}\left[\phi_k \zeta E_t \widetilde{u}_{t+1} - \widetilde{r}_t^p + \widetilde{\nu}_t\right] - \frac{\phi_q}{\kappa} E_t \widetilde{\nu}_{t+1},$$

where lowercase letters with a tilde denote the respective log deviations of the variables from their steady state, $\widetilde{r}_t^p$ denotes the log-deviation of the ex-ante real interest rate from steady state, and $\phi_q = (1-\delta)\beta$ and $\phi_k = 1 - \phi_q$.

A.1 Derivation of equation (3)

The first-order conditions are

$$
\begin{aligned}
I_t: \quad 1 =& \nu_t Q_t \left[1 - \mathcal{S}\left(\frac{I_t}{I_{t-1}}\right) - \mathcal{S}'\left(\frac{I_t}{I_{t-1}}\right)\left(\frac{I_t}{I_{t-1}}\right)\right] \\
&+ \beta E_t \left\{\frac{\lambda_{t+1}}{\lambda_t}\mathcal{S}'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2 \nu_{t+1} Q_{t+1}\right\}, \\
\hat{K}_{t+1}: \quad Q_t =& \beta E_t \left\{\frac{\lambda_{t+1}}{\lambda_t}\left[r^k_{t+1} u_{t+1} - a(u_{t+1}) + Q_{t+1}(1-\delta)\right]\right\}, \\
B_{t+1}: \quad 1 =& \beta E_t \left\{\frac{\lambda_{t+1}}{\lambda_t}\frac{R_t}{\pi_{t+1}}\right\}, \text{ and} \\
u_t: \quad r^k_t =& a'(u_t),
\end{aligned}
$$

where Q_t denotes the marginal $\mathcal{Q}$, defined as the ratio of the Lagrange multipliers associated with the capital accumulation equation and the budget constraint (λ_t), and π_{t+1} is the inflation rate in period $t+1$.

Log-linearizing the above first-order conditions around the non-stochastic steady state yields

$$
\widetilde{q}_t = \kappa\left(\widetilde{i}_t - \widetilde{i}_{t-1}\right) - \beta\kappa\left(E_t\widetilde{i}_{t+1} - \widetilde{i}_t\right) - \widetilde{\nu}_t, \tag{CS 6}
$$

$$
\widetilde{q}_t = E_t\widetilde{\lambda}_{t+1} - \widetilde{\lambda}_t + \beta\bar{r}^k E_t\widetilde{r}^k_{t+1} + \beta(1-\delta)E_t\widetilde{q}_{t+1}, \tag{CS 7}
$$

$$
E_t\widetilde{\lambda}_{t+1} - \widetilde{\lambda}_t = -(\widetilde{r}_t - E_t\widetilde{\pi}_{t+1}), \text{ and} \tag{CS 8}
$$

$$
\widetilde{r}^k_t = \zeta\widetilde{u}_t, \tag{CS 9}
$$

where lowercase letters with a tilde denote the respective log deviations of the variables from their steady state. Although (CS 6)-(CS 9) can be estimated, the empirical literature has struggled to find an appropriate proxy for $\tilde{q}_t$, the marginal $\mathcal{Q}$, which is unobservable. Hayashi (1982) showed that under some regularity conditions the average $\mathcal{Q}$ is equivalent to marginal $\mathcal{Q}$. However, subsequent empirical studies have confirmed such regularity conditions to be unsatisfactory, finding insignificant coefficients on average $\mathcal{Q}$. Thus, we follow the treatment in Groth and Khan (2010) and get rid of $\tilde{q}_t$ from the log-linearized conditions. Substituting

(CS 6) and (CS 8) into (CS 7) yields our preferred baseline investment Euler equation with IAC

$$\begin{aligned}\widetilde{i}_t =& \frac{1/\kappa}{1+\beta+\phi_q}\left[\phi_k E_t \widetilde{r}^k_{t+1} - E_t \widetilde{r}^p_t + \widetilde{\nu}_t\right] \\ &+ \frac{1}{1+\beta+\phi_q}\widetilde{i}_{t-1} + \frac{\beta+\phi_q(1+\beta)}{1+\beta+\phi_q}E_t\widetilde{i}_{t+1} - \frac{\beta\phi_q}{1+\beta+\phi_q}E_t\widetilde{i}_{t+2} - \frac{\phi_q/\kappa}{1+\beta+\phi_q}E_t\widetilde{\nu}_{t+1},\end{aligned} \tag{CS 10}$$

where $\widetilde{r}^p_t$ denotes the log-deviation of ex-ante real interest rate from steady state, i.e., $\widetilde{r}^p_t = \widetilde{r}_t - \widetilde{\pi}_{t+1}$, and $\phi_q = (1-\delta)/(1-\delta+\bar{r}^k)$; $\phi_k = \bar{r}^k/(1-\delta+\bar{r}^k)$; and $\bar{r}^k = 1/\beta - 1 + \delta$.[1]

We can use (CS 9) to substitute out the rental rate of capital, $\widetilde{r}^k_t$, which is an unobservable variable, with the capacity utilization, $\widetilde{u}_t$, for which a time series is available. So, (CS 10) becomes

$$\begin{aligned}\widetilde{i}_t =& \frac{1/\kappa}{1+\beta+\phi_q}\left[\phi_k \zeta E_t \widetilde{u}_{t+1} - E_t \widetilde{r}^p_t + \widetilde{\nu}_t\right] \\ &+ \frac{1}{1+\beta+\phi_q}\widetilde{i}_{t-1} + \frac{\beta+\phi_q(1+\beta)}{1+\beta+\phi_q}E_t\widetilde{i}_{t+1} - \frac{\beta\phi_q}{1+\beta+\phi_q}E_t\widetilde{i}_{t+2} - \frac{\phi_q/\kappa}{1+\beta+\phi_q}E_t\widetilde{\nu}_{t+1}.\end{aligned}$$

Note that this equation is the same as equation (2) in the main text, that is simply rewritten in first differences of the investment terms. For any variable x_t the rational expectations (RE) forecast error is $\eta^x_{t|t-1} = x_t - E_{t-1}(x_t)$, which implies that $E_t(x_{t+1}) = x_{t+1} - \eta^x_{t+1|t}$. Moreover, $E_t\widetilde{\nu}_{t+1} = \rho\widetilde{\nu}_t$, since $\widetilde{\nu}_t \sim AR(1) : \widetilde{\nu}_t = \rho\widetilde{\nu}_{t-1} + \varepsilon^v_t$. Finally, define $E_t(\widetilde{i}_{t+2}) = \widetilde{i}_{t+2} - \eta^i_{t+2|t}$, then

$$\begin{aligned}\widetilde{i}_t =& \frac{1/\kappa}{1+\beta+\phi_q}\left[\phi_k\zeta\widetilde{u}_{t+1} - \phi_k\zeta\eta^u_{t+1|t} - \widetilde{r}^p_t + \eta^\pi_{t+1|t}\right] + \frac{1}{1+\beta+\phi_q}\widetilde{i}_{t-1} \\ &+ \frac{\beta+\phi_q(1+\beta)}{1+\beta+\phi_q}\left(\widetilde{i}_{t+1} - \eta^i_{t+1|t}\right) - \frac{\beta\phi_q}{1+\beta+\phi_q}\left(\widetilde{i}_{t+2} - \eta^i_{t+2|t}\right) + \frac{1/\kappa\,(1-\phi_q\rho)}{1+\beta+\phi_q}\widetilde{\nu}_t,\end{aligned}$$

[1] Replacing $\bar{r}^k$ into the equations of ϕ_q and ϕ_k results in $\phi_q = \beta(1-\delta)$ and $\phi_k = 1-\beta(1-\delta)$ respectively.

or

$$
\begin{aligned}
\widetilde{i}_t = & \frac{1/\kappa}{1+\beta+\phi_q}\left[\phi_k \zeta \widetilde{u}_{t+1} - \widetilde{r}_t^p\right] + \frac{1}{1+\beta+\phi_q}\widetilde{i}_{t-1} + \frac{\beta+\phi_q(1+\beta)}{1+\beta+\phi_q}\widetilde{i}_{t+1} \\
& - \frac{\beta\phi_q}{1+\beta+\phi_q}\widetilde{i}_{t+2} + \varepsilon_t,
\end{aligned} \tag{CS 11}
$$

where $\varepsilon_t = \frac{1/\kappa}{1+\beta+\phi_q}\left[-\phi_k\zeta\eta^u_{t+1|t} + \eta^\pi_{t+1|t}\right] - \frac{\beta+\phi_q(1+\beta)}{1+\beta+\phi_q}\eta^i_{t+1|t} + \frac{\beta\phi_q}{1+\beta+\phi_q}\eta^i_{t+2|t} + \frac{1/\kappa(1-\phi_q\rho)}{1+\beta+\phi_q}\widetilde{\nu}_t$.

Using the facts that $\widetilde{i}_t = \frac{1}{1+\beta+\phi_q}\widetilde{i}_t + \frac{\beta+\phi_q}{1+\beta+\phi_q}\widetilde{i}_t$, and $\frac{\beta+\phi_q(1+\beta)}{1+\beta+\phi_q}\widetilde{i}_{t+1} = \frac{\beta+\phi_q}{1+\beta+\phi_q}\widetilde{i}_{t+1} + \frac{\beta\phi_q}{1+\beta+\phi_q}\widetilde{i}_{t+1}$, the terms in $\widetilde{i}$ could be written as first difference, so equation (CS 11) becomes

$$
\frac{1}{1+\beta+\phi_q}\Delta\widetilde{i}_t = \frac{1/\kappa}{1+\beta+\phi_q}\left[\phi_k\zeta\widetilde{u}_{t+1} - \widetilde{r}^p_t\right] + \frac{\beta+\phi_q}{1+\beta+\phi_q}\Delta\widetilde{i}_{t+1} - \frac{\beta\phi_q}{1+\beta+\phi_q}\Delta\widetilde{i}_{t+2} + \varepsilon_t,
$$

or

$$
\Delta\widetilde{i}_t = \frac{\phi_k}{\kappa}\zeta\widetilde{u}_{t+1} - \frac{1}{\kappa}\widetilde{r}^p_t + (\beta+\phi_q)\,\Delta\widetilde{i}_{t+1} - \beta\phi_q\Delta\widetilde{i}_{t+2} + (1+\beta+\phi_q)\,\varepsilon_t. \tag{CS 12}
$$

We then just eliminate $\widetilde{\nu}_t$ in the error term ε_t, again by lagging (CS 12), multiplying it by ρ, which results in

$$
\rho\Delta\widetilde{i}_{t-1} = \frac{\rho\phi_k}{\kappa}\zeta\widetilde{u}_t - \frac{\rho}{\kappa}\widetilde{r}^p_{t-1} + \rho\,(\beta+\phi_q)\,\Delta\widetilde{i}_t - \rho\beta\phi_q\Delta\widetilde{i}_{t+1} + \rho\,(1+\beta+\phi_q)\,\varepsilon_{t-1},
$$

and subtracting the result from (CS 12), such that

$$
\begin{aligned}
\Delta\widetilde{i}_t - \rho\Delta\widetilde{i}_{t-1} = & \frac{\phi_k}{\kappa}\zeta\left(\widetilde{u}_{t+1} - \rho\widetilde{u}_t\right) - \frac{1}{\kappa}\left(\widetilde{r}^p_t - \rho\widetilde{r}^p_{t-1}\right) + (\beta+\phi_q)\left(\Delta\widetilde{i}_{t+1} - \rho\Delta\widetilde{i}_t\right) \\
& - \beta\phi_q\left(\Delta\widetilde{i}_{t+2} - \rho\Delta\widetilde{i}_{t+1}\right) + (1+\beta+\phi_q)\left(\varepsilon_t - \rho\varepsilon_{t-1}\right).
\end{aligned}
$$

Rearranging terms we obtain the baseline specification (3)

$$
\begin{aligned}
\left[1+\rho\left(\beta+\phi_q\right)\right]\Delta\widetilde{i}_t = & \rho\Delta\widetilde{i}_{t-1} + \left(\beta+\phi_q+\rho\beta\phi_q\right)\Delta\widetilde{i}_{t+1} - \beta\phi_q\Delta\widetilde{i}_{t+2} \\
& + \frac{\phi_k}{\kappa}\zeta\widetilde{u}_{t+1} - \frac{\rho\phi_k}{\kappa}\zeta\widetilde{u}_t - \frac{1}{\kappa}\widetilde{r}^p_t + \frac{\rho}{\kappa}\widetilde{r}^p_{t-1} + \epsilon_t,
\end{aligned}
$$

where

$$
\epsilon_t := (1+\beta+\phi_q)\left(\varepsilon_t - \rho\varepsilon_{t-1}\right), \tag{CS 13}
$$

$(\varepsilon_t - \rho\varepsilon_{t-1}) = \frac{1/\kappa}{1+\beta+\phi_q}\left[-\phi_k\eta^u_{t+1|t} + \eta^\pi_{t+1|t} - \rho\left(-\phi_k\eta^u_{t|t-1} + \eta^\pi_{t|t-1}\right)\right] - \frac{\beta+\phi_q(1+\beta)}{1+\beta+\phi_q}\left(\eta^i_{t+1|t} - \rho\eta^i_{t|t-1}\right)$ $+\frac{\beta\phi_q}{1+\beta+\phi_q}\left(\eta^i_{t+2|t} - \rho\eta^i_{t+1|t}\right) + \frac{1/\kappa(1-\phi_q\rho)}{1+\beta+\phi_q}\varepsilon^v_t$, and $\varepsilon^v_t = \widetilde{\nu}_t - \rho\widetilde{\nu}_{t-1}$.

B Computational Details

The empirical moments of the linear model can be represented by $\frac{1}{T}\sum_{t=1}^T f_t(\theta, d)$, where $f_t(\theta, d) = Z_t'(Y_t b(\theta) - X_t d)$, $Y_t = \begin{bmatrix} \Delta\widetilde{i}_t & \Delta\widetilde{i}_{t-1} & \Delta\widetilde{i}_{t+1} & \Delta\widetilde{i}_{t+2} & \widetilde{r}^p_t & \widetilde{r}^p_{t-1} & \widetilde{u}_t & \widetilde{u}_{t+1} \end{bmatrix}$, $Z_t = (X_t, Z_{2,t})$ is the set of instrumental variables partitioned into included (X_t) and excluded $(Z_{2,t})$ instruments, $b(\theta)' = \left[1 + \rho(\beta + \phi_q), -\rho, -(\beta + \phi_q + \rho\beta\phi_q), \beta\phi_q, \frac{1}{\kappa}, -\frac{\rho}{\kappa}, \phi_k\frac{\rho\zeta}{\kappa}, -\phi_k\frac{\zeta}{\kappa}\right]'$ is a vector which contains the structural parameters and d are the strongly identified parameters, which are estimated before the computation of the statistical tests. The variable X_t is 1 corresponding to the constant in the estimated regression specification, which captures all the steady-state terms. We use $Z_{2,t} = \left\{\Delta i_{t-1}, r^p_{t-2}, u_{t-1}\right\}$ in our baseline results. The sample size is T.

B.1 S and qLL-S tests

Under $H_0 : \theta = \theta_0$, $b(\theta_0)$ is fixed. The S statistic is

$$S(\theta_0) = \min_d \frac{1}{T}\sum_{t=1}^T f_t(\theta_0, d)' \widehat{V}(\theta_0, d)^{-1} \sum_{t=1}^T f_t(\theta_0, d). \qquad \text{(CS 14)}$$

The minimand in the above expression is the so-called continuously updated GMM objective function, evaluated at the continuously updated estimator for the untested parameter d under H_0, see Stock and Wright (2000). The variance estimator $\widehat{V}(\theta_0, d)$ is a heteroskedasticity and autocorrelation consistent (HAC) estimator of Var $\left(\frac{1}{\sqrt{T}}\sum_{t=1}^T f_t(\theta_0, d)\right)$

$$\widehat{V} = \widehat{\Gamma}_0 + \sum_{j=1}^T \omega_j\left(\widehat{\Gamma}_j + \widehat{\Gamma}_j'\right),$$

where $\widehat{\Gamma}_j = \left[\frac{1}{T}\sum_{t=j+1}^T \widehat{w}_t\widehat{w}_t'\right]$, $\widehat{w}_t$ is $f_t\left(\theta_0, \hat{d}\right) - \bar{f}_T\left(\theta_0, \hat{d}\right)$, $\bar{f}_T\left(\theta_0, \hat{d}\right) = \frac{1}{T}\sum_{t=1}^T f_t\left(\theta_0, \hat{d}\right)$. The parameter ω_j represents the Barlett kernel.

The S statistic is obtained by plugging $\hat{d}(\theta_0)$ into the objective function (CS 14). The S test at level α rejects $H_0 : \theta = \theta_0$ when the S statistic exceeds the $1-\alpha$ quantile of the χ^2 distribution with $k_z - k_x$ degrees of freedom, where k_z and k_x are the number of elements in vectors Z_t and X_t, respectively.

The qLL-S test rejects for large values of the statistic

$$\text{qLL–S}(\theta_0) = \frac{10}{11}\text{S}(\theta_0) + \text{qLL–S}^B(\theta_0)$$

where $\text{S}(\theta_0)$ is the S statistic evaluated at $\theta = \theta_0$, and $\text{qLL–S}^B(\theta_0)$ is the statistic that detects violations of the moment conditions in subsamples. The algorithm for computing the qLL–S^B is detailed in Magnusson and Mavroeidis (2014), where one can also find tables of critical values.

The confidence sets derived from the tests are obtained by performing a grid search over the parameter space. The 90% confidence sets are formed by the collection of points that do not reject $H_0 : \theta = \theta_0$ at 10% significance level.

B.2 Split-sample S test

We derive a GMM version of the split-sample Anderson-Rubin test proposed by Mikusheva (2021) for linear models. Let $\bar{Y}_t$ be the demeaned values of Y_t. Define $\bar{W}_t(\theta) = \bar{Y}_t \frac{\partial b(\theta)}{\partial \theta'}$, which is of dimension 1×3, and let $\mathbf{W}(\theta)$ be the $T \times 3$ matrix with stacked terms $\bar{W}_t(\theta)$, $t = 1, \ldots, T$. Define also the matrices $\mathbf{Y}$ and $\mathbf{Z}$ of dimensions $T \times 8$ and $T \times k$ ($k = 3$ in the baseline case) of stacked elements of $\bar{Y}_t$ and demeaned excluded instruments $\bar{Z}_{2,t}$. Partition $\mathbf{W}(\theta) = \mathbf{Y}\frac{\partial b(\theta)}{\partial \theta'}$, $\mathbf{Y}$ and $\mathbf{Z}$ as $\mathbf{W}(\theta) = [\mathbf{W}_1(\theta) : \mathbf{W}_p(\theta) : \mathbf{W}_2(\theta)]$, $\mathbf{Y} = [\mathbf{Y}_1 : \mathbf{Y}_p : \mathbf{Y}_2]$, and $\mathbf{Z} = [\mathbf{Z}_1 : \mathbf{Z}_p : \mathbf{Z}_2]$.

In our case, the first subsample corresponds to 45% of the initial observations. The terms $\mathbf{W}_p(\theta)$ and $\mathbf{Z}_p$ are not used in the procedure in order to keep the exogeneity assumption valid. Following Mikusheva (2021, p. 30), we set $p = 3$ because the error ϵ_t in (CS 13) is adapted to the $t+2$ information set and the instruments include variables dated $t-1$. Then, estimate the fitted value of $\mathbf{W}_2(\theta)$ as $\hat{\mathbf{W}}_2(\theta) = \mathbf{Z}_2\hat{\pi}_1(\theta)$, where $\hat{\pi}_1(\theta) = (\mathbf{Z}_1'\mathbf{Z}_1)^{-1}\mathbf{Z}_1'\mathbf{W}_1(\theta)$ and $\mathbf{W}_1(\theta) = \mathbf{Y}_1\frac{\partial b(\theta)}{\partial \theta'}$.

Finally, we compute the split-sample S statistic as

$$S_M(\theta) = \frac{1}{T_2} b(\theta)' \mathbf{Y}_2' \hat{\mathbf{W}}_2(\theta) \left[\widehat{\Omega}(\theta)\right]^{-1} \hat{\mathbf{W}}_2(\theta)' \mathbf{Y}_2 b(\theta),$$

where $\widehat{\Omega}(\theta)$ is the HAC estimator of the variance of $\frac{1}{\sqrt{T_2}} \sum_{t=t_2}^{T} \hat{\bar{W}}_t(\theta)' \bar{Y}_t b(\theta)$ and T_2 corresponds to the number of observations of the last subsample.

The split-sample S test at level α rejects $H_0 : \theta = \theta_0$ when $S_M(\theta_0)$ exceeds the $1-\alpha$ quantile of a χ^2 distribution with 3 degrees of freedom.

C Data

C.1 Data Sources for baseline analysis

- **Gross Private Domestic Investment** [GPDI]**:** Billions of Dollars, Seasonally Adjusted Annual Rate; Source: U.S. Bureau of Economic Analysis; FRED - https://fred.stlouisfed.org/series/GPDI.

- **Fixed Private Investment** [FPI]**:** Billions of Dollars, Seasonally Adjusted Annual Rate; Source: U.S. Bureau of Economic Analysis; FRED - https://fred.stlouisfed.org/series/FPI.

- **Personal Consumption Expenditures: Durable Goods** [PCDG]**:** Billions of Dollars, Seasonally Adjusted Annual Rate; Source: U.S. Bureau of Economic Analysis; FRED - https://fred.stlouisfed.org/series/PCDG.

- **Gross Domestic Product (implicit price deflator)** [GDPDEF]**:** Index 2012=100, Seasonally Adjusted; Source: U.S. Bureau of Economic Analysis; FRED - https://fred.stlouisfed.org/series/GDPDEF.

- **Gross Private Domestic Investment (implicit price deflator)** [A006RD3Q086SBEA]**:** Index 2012=100, Seasonally Adjusted; Source: U.S. Bureau of Economic Analysis; FRED - https://fred.stlouisfed.org/series/A006RD3Q086SBEA.

- **Gross Private Domestic Investment: Fixed Investment (implicit price deflator)** [A007RD3Q086SBEA]**:** Index 2012=100, Seasonally Adjusted; Source: U.S. Bureau of Economic Analysis; FRED - https://fred.stlouisfed.org/series/A007RD3Q086SBEA#0.

- **Personal Consumption Expenditures: Durable goods (implicit price deflator)** [DDURRD3Q086SBEA]**:** Index 2012=100, Seasonally Adjusted; Source: U.S. Bureau of Economic Analysis; FRED - https://fred.stlouisfed.org/series/DDURRD3Q086SBEA.

- **Effective Federal Funds Rate** [FEDFUNDS]**:** Percent, Not Seasonally Adjusted; Source: Board of Governors of the Federal Reserve System; FRED - https://fred.stlouisfed.org/series/FEDFUNDS.

- **Capacity Utilization: Total Index** [TCU]**:** Percent of Capacity, Seasonally Adjusted; Source: Board of Governors of the Federal Reserve System; FRED - https://fred.stlouisfed.org/series/TCU#0.

C.2 Additional exogenous instruments

- **Romer and Romer (2004)'s narrative-based monetary policy shock** (1969m3-2007m12); retrieved from Valerie A. Ramey's website under *Data and Programs for "Macroeconomic Shocks and Their Propagation", 2016 Handbook of Macroeconomics.* - https://econweb.ucsd.edu/~vramey/research.html#data.

- **Ramey and Zubairy (2018)'s military news shock** (1967q1-2015q4); retrieved from Valerie A. Ramey's website under *Programs and Data for "Government Spending Multipliers in Good Times and in Bad" with Sarah Zubairy, April 2018 Journal of Political Economy.* - https://econweb.ucsd.edu/~vramey/research.html#data.

- **Spot Crude Oil Price: West Texas Intermediate (WTI)** [WTISPLC] (1967q1-2019q4); Deflated using CPI, Not Seasonally Adjusted; Source: Federal Reserve Bank of St. Louis; FRED - https://fred.stlouisfed.org/series/WTISPLC.

- **VXO** (1967q1-2019q4); Source: Chicago Board of Options Exchange (CBOE) and retrieved from FRED - https://fred.stlouisfed.org/series/VXOCLS.

  Note: This index is unavailable before 1986. Following Bloom (2009), pre-1986 monthly return volatilities are computed as the monthly standard deviation of the daily S&P500 index normalized to the same mean and variance as the VXO index when they overlap from 1986 onward.

C.3 Data Transformation

Investment: Investment series is first divided by the civilian non-institutional population (16 years or over) to convert into per capita terms and the resulting per capita series is then deflated using the respective implicit price deflators. Two per capita measures of investment are used in the analysis. They are:

1. SW - Real Fixed Private Investment (FPI).

2. JPT - sum of Real Gross Private Domestic Investment (GPDI) and Real Personal Consumption Expenditure: Durables Goods (PCDG).

The investment measures are computed, respectively, as $\frac{FPI}{P_{fpi}}$ and $\frac{GDPI}{P_{gpdi}} + \frac{PCDG}{P_{pcdg}}$, where P_{gpdi}, P_{pcdg} and P_{fpi} are the respective implicit price deflators. Growth rates of investment are then computed as the log difference of the resulting series.

Inflation: Log difference of the quarterly implicit GDP price deflator.

Real (ex-post) interest rate: Difference between the Federal Funds Rate and the GDP deflator inflation rate.

Capacity utilization: Log of the capacity utilization index.

Narrative-based monetary policy shock: Quarterly average of the monthly series from Romer and Romer (2004).

Narrative-based military news shock: Defense news variable of Ramey (2016) scaled by trend GDP following Ramey and Zubairy (2018).

VXO: Quarterly average of the monthly series, demeaned and standardized.

Oil: Log difference of the real oil price series.

D Robustness checks

In this section we report further results to investigate the robustness of the empirical results reported in the main text. Figure S.1 shows the results when we use two lags of endogenous variables as instruments and compares them with our baseline results using one lag. Figure S.2 shows the results when we restrict our estimation sample to 2004Q4 (as in SW and JPT) and compares it with our baseline sample ending in 2019Q4. Figures S.3 and S.4 correspond to Figures 5 and 6 in the main text, respectively, but the external instruments are now added together with r^p_{t-2} and u_{t-1} in the set of instruments. The results are unchanged when using external instruments both for the SW and JPT investment measures, respectively. The main conclusion from these sensitivity analyses is that the results reported in the paper remain largely robust.

E The Capital Adjustment Cost Model

In this section, we derive the investment Euler equation with capital adjustment cost. Similar to the capital accumulation equation (1), the representative household accumulates end-of-period t capital

$$\hat{K}_{t+1} = \nu_t I_t + (1-\delta)\hat{K}_t - D(\hat{K}_t, I_t). \qquad \text{(CS 15)}$$

The function $D(\hat{K}_t, I_t)$ is the capital adjustment cost (CAC) which can be defined as

$$D(\hat{K}_t, I_t) = \frac{\sigma}{2}\left(\frac{I_t}{\hat{K}_t} - \delta\right)^2 \hat{K}_t,$$

where $\sigma > 0$ governs the magnitude of adjustment costs to capital accumulation and δ is the depreciation rate. This functional form is a variant of the one considered in Lucas (1967) and Lucas and Prescott (1971), and has reappeared more recently in the DSGE literature, see, for example, Christiano, Eichenbaum, and Rebelo (2011) and Basu and Bundick (2017). The representative household still chooses I_t, $\hat{K}_{t+1}$, B_{t+1}, and u_t to maximise (**??**) under

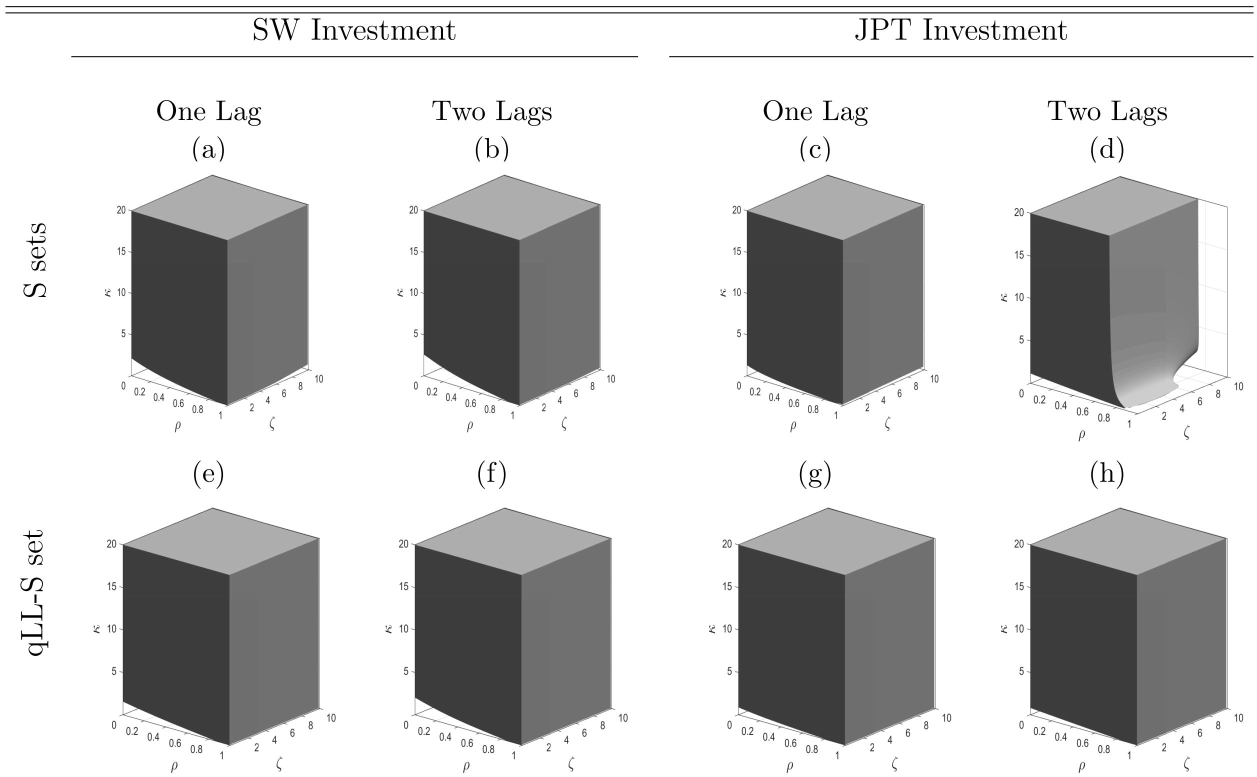

Figure S.1: 90% S and qLL-S confidence sets for $\theta = (\rho, \kappa, \zeta)$ in the investment euler equation model (3). Instruments: One lag - constant, Δi_{t-1}, r^p_{t-2}, u_{t-1}; Two lags - constant, Δi_{t-1}, Δi_{t-2}, r^p_{t-2}, r^p_{t-3}, u_{t-1}, u_{t-2}. Left two columns show the results based on using Fixed Private Investment as investment proxy, while right two columns use the sum of Gross Private Domestic Investment and Personal Consumption Expenditure on Durable Goods as investment proxy. Period: 1967Q1-2019Q4. Newey and West (1987) HAC.

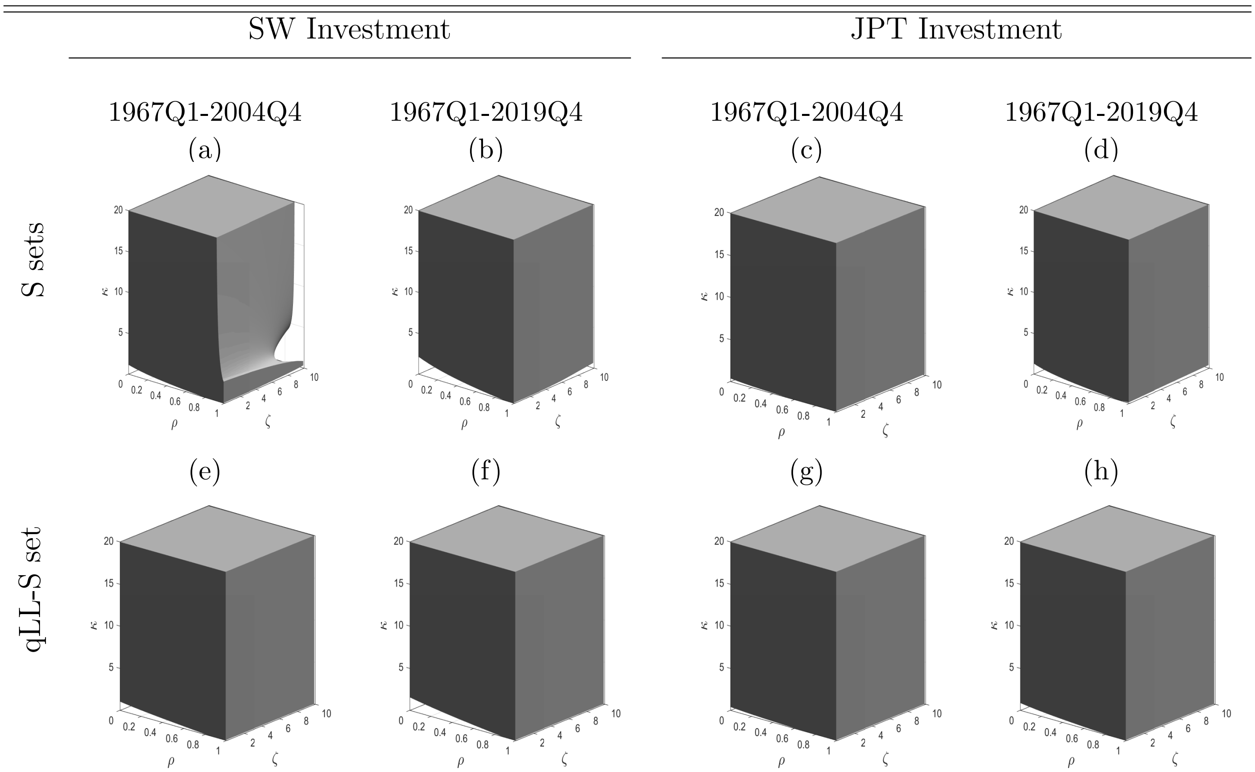

Figure S.2: 90% S and qLL-S confidence sets for $\theta = (\rho, \kappa, \zeta)$ in the investment euler equation model (3). Instruments: constant, Δi_{t-1}, r^{p}_{t-2}, u_{t-1}. Left two columns show the results based on using Fixed Private Investment as investment proxy, while right two columns use the sum of Gross Private Domestic Investment and Personal Consumption Expenditure on Durable Goods as investment proxy. Newey and West (1987) HAC.

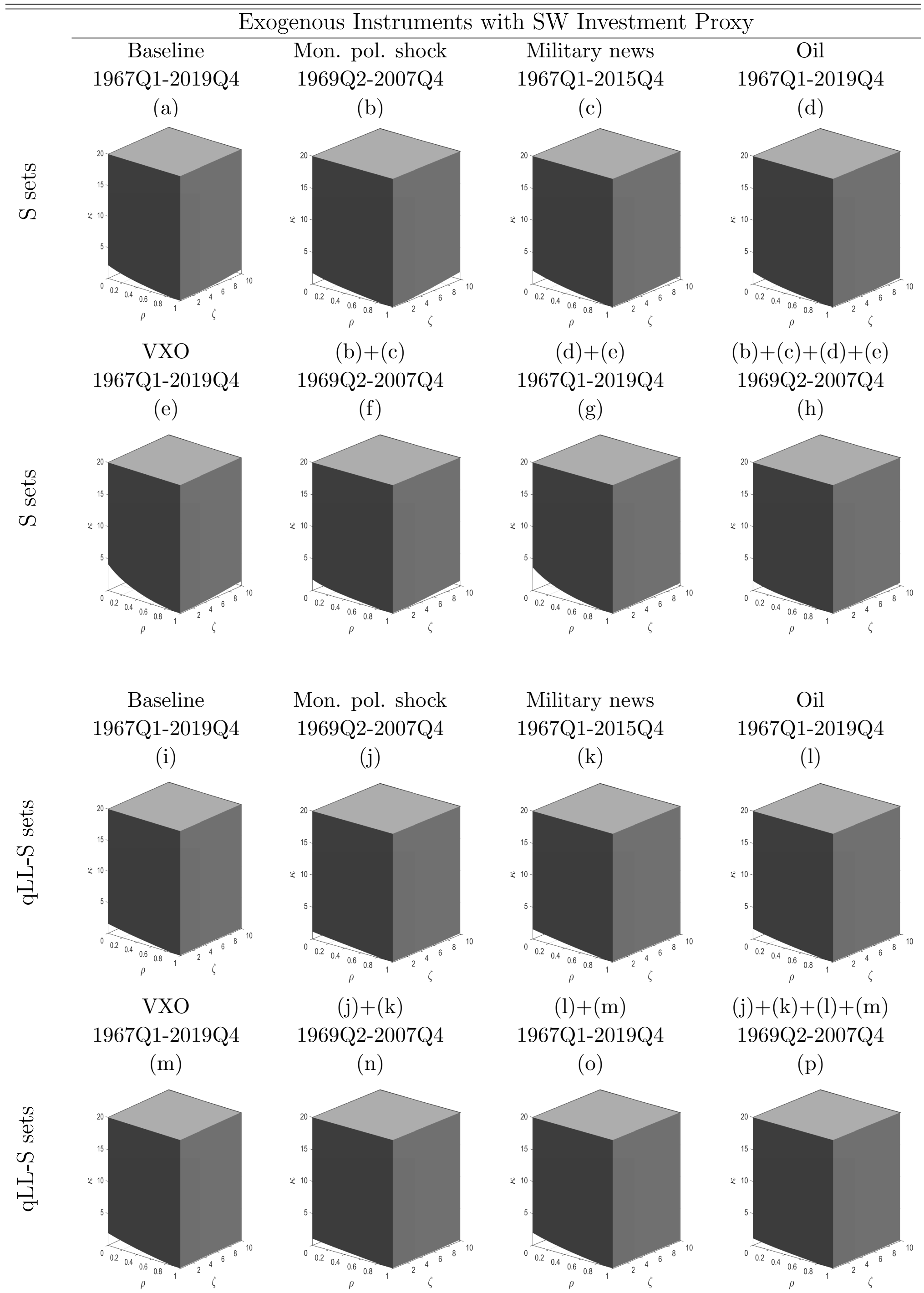

Figure S.3: 90% S and qLL-S confidence sets for $\theta = (\rho, \kappa, \zeta)$ derived from the investment Euler equation model (3) using Fixed Private Investment as investment proxy. A constant, Δi_{t-1}, r^p_{t-2}, and u_{t-1} are common instruments in all specifications. The additional instrument(s) by specification is (are): Mon. pol. shock: Romer and Romer's (2004) monetary policy shock; Military news: Ramey and Zubairy's (2018) military news shock; Oil: growth rate of real oil price; VXO: financial uncertainty.

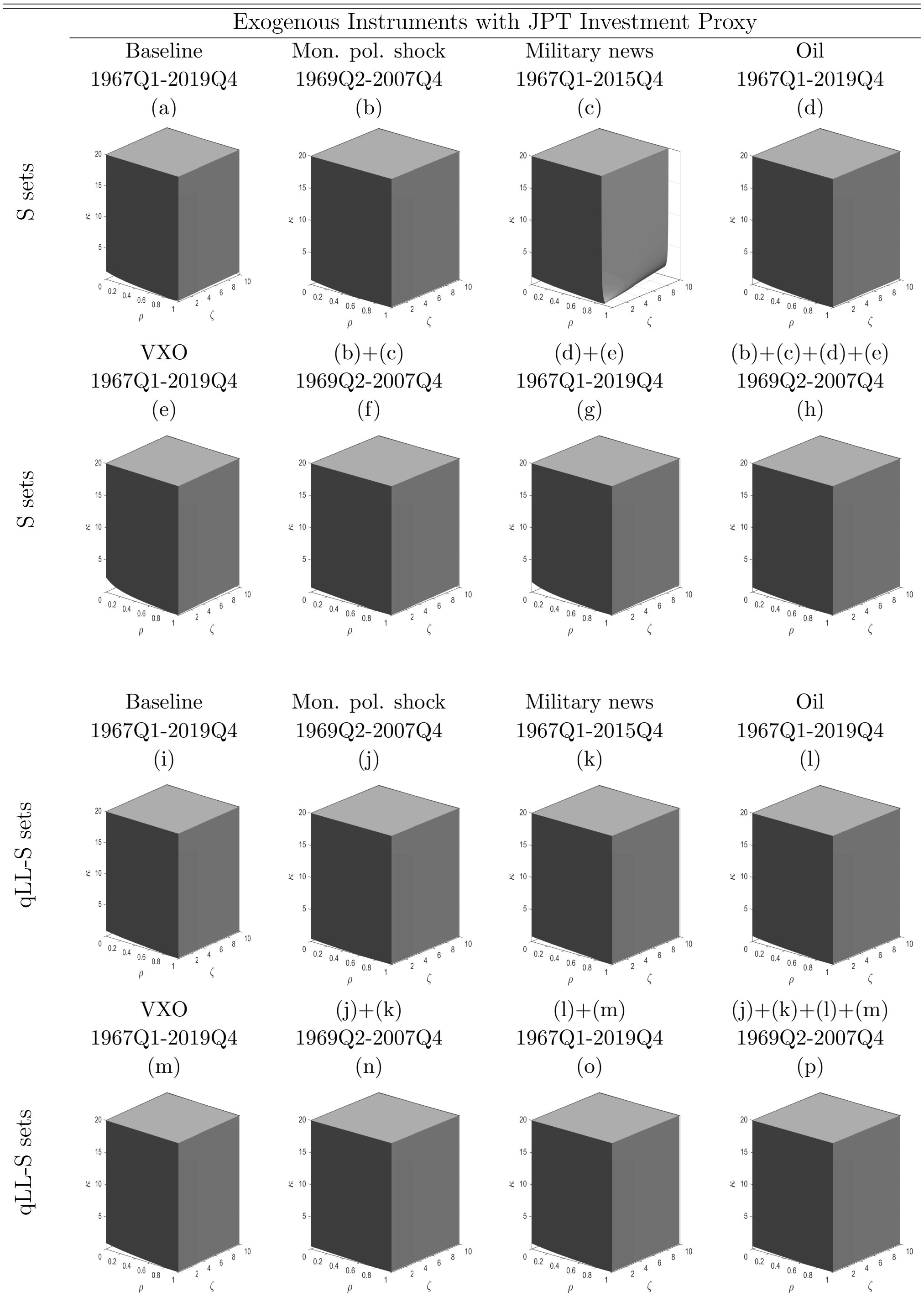

Figure S.4: 90% S and qLL-S confidence sets for $\theta = (\rho, \kappa, \zeta)$ derived from the investment Euler equation model (3) using the sum of Gross Private Domestic Investment and Personal Consumption Expenditure on Durable Goods as investment proxy. A constant, Δi_{t-1}, r^p_{t-2}, and u_{t-1} are common instruments in all specifications. The additional instrument(s) by specification is (are): Mon. pol. shock: Romer and Romer's (2004) monetary policy shock; Military news: Ramey and Zubairy's (2018) military news shock; Oil: growth rate of real oil price; VXO: financial uncertainty.

the period-by-period budget constraint (CS 2) and capital accumulation equation (CS 15). The relevant first-order conditions required to derive the log-linearized investment Euler equations are

$$\begin{aligned}
I_t: \quad \nu_t Q_t &= \left[1+\sigma\left(\frac{I_t}{\hat{K}_t}-\delta\right)Q_t\right], \\
\hat{K}_{t+1}: \quad Q_t &= \beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[r^k_{t+1}u_{t+1}-a\left(u_{t+1}\right)-\frac{\sigma}{2}\left(\frac{I_{t+1}}{\hat{K}_{t+1}}-\delta\right)^2\right]\right\} \\
&+\beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[\sigma\left(\frac{I_{t+1}}{\hat{K}_{t+1}}-\delta\right)\frac{I_{t+1}}{\hat{K}_{t+1}}+Q_{t+1}\left(1-\delta\right)\right]\right\}, \\
B_{t+1}: \quad 1 &= \beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\frac{R_t}{\pi_{t+1}}\right\}, \text{ and} \\
u_t: \quad r^k_t &= a'\left(u_t\right),
\end{aligned}$$

where Q_t denotes the marginal $\mathcal{Q}$, defined as the ratio of the Lagrange multipliers associated with the capital accumulation equation and the budget constraint (λ_t), and π_{t+1} is the inflation rate in period $t+1$.

Log-linearization of the FOCs around the steady state yields

$$\widetilde{q}_t = \sigma\delta\left(\widetilde{i}_t-\widetilde{k}_t\right)-\widetilde{\nu}_t, \tag{CS 16}$$

$$\widetilde{q}_t = E_t\widetilde{\lambda}_{t+1}-\widetilde{\lambda}_t+\beta\sigma\delta^2\left(E_t\widetilde{i}_{t+1}-\widetilde{k}_{t+1}\right)+\beta(1-\delta)E_t\widetilde{q}_{t+1}+\beta\bar{r}^k E_t\widetilde{r}^k_{t+1}, \tag{CS 17}$$

$$E_t\widetilde{\lambda}_{t+1}-\widetilde{\lambda}_t = -(\widetilde{r}_t-E_t\widetilde{\pi}_{t+1}), \text{ and} \tag{CS 18}$$

$$\widetilde{r}^k_t = \zeta\widetilde{u}_t. \tag{CS 19}$$

Similar to the IAC model, combining (CS 16)-(CS 18) yields our baseline investment Euler equation with CAC

$$\widetilde{i}_t = \widetilde{k}_t+\frac{1}{\sigma\delta}\left[\beta\bar{r}^k E_t\widetilde{r}^k_{t+1}-E_t\widetilde{r}^p_t\right]+\beta\left(E_t\widetilde{i}_{t+1}-\widetilde{k}_{t+1}\right)+\frac{1}{\sigma\delta}\widetilde{\nu}_t-\frac{\beta\left(1-\delta\right)}{\sigma\delta}E_t\widetilde{\nu}_{t+1}. \tag{CS 20}$$

Using equation (CS 19), we replace $\widetilde{r}^k_{t+1}$ by $\zeta\widetilde{u}_{t+1}$ to obtain

$$\widetilde{i}_t = \widetilde{k}_t+\frac{1}{\sigma\delta}\left[\beta\bar{r}^k\zeta E_t\widetilde{u}_{t+1}-E_t\widetilde{r}^p_t\right]+\beta\left(E_t\widetilde{i}_{t+1}-\widetilde{k}_{t+1}\right)+\frac{1}{\sigma\delta}\widetilde{\nu}_t-\frac{\beta\left(1-\delta\right)}{\sigma\delta}E_t\widetilde{\nu}_{t+1}.$$

As before, we replace the variables in expectations with the observed values and its respective rational expectation forecast error and also replace $E_t\widetilde{\nu}_{t+1}$ by $\rho\widetilde{\nu}_t$, which results in

$$\widetilde{i}_t = \widetilde{k}_t + \frac{1}{\sigma\delta}\left[\beta\bar{r}^k\zeta\left(\widetilde{u}_{t+1} - \eta^u_{t+1|t}\right) - (\widetilde{r}^p_t + \eta^\pi_{t+1|t})\right] + \beta\left(\widetilde{i}_{t+1} - \eta^i_{t+1|t} - \widetilde{k}_{t+1}\right) + \frac{1-\beta\left(1-\delta\right)\rho}{\sigma\delta}\widetilde{\nu}_t,$$

or

$$\widetilde{i}_t = \widetilde{k}_t + \frac{1}{\sigma\delta}\left[\beta\bar{r}^k\zeta\widetilde{u}_{t+1} - \widetilde{r}^p_t\right] + \beta\left(\widetilde{i}_{t+1} - \widetilde{k}_{t+1}\right) + \varepsilon_t, \qquad \text{(CS 21)}$$

where $\varepsilon_t = -\frac{1}{\sigma\delta}\left[\beta\bar{r}^k\zeta\eta^u_{t+1|t} + \eta^\pi_{t+1|t}\right] - \beta\eta^i_{t+1|t} + \frac{1-\beta(1-\delta)\rho}{\sigma\delta}\widetilde{\nu}_t$. Since in steady state $\overline{I} = \delta\overline{K}$ and $\overline{\nu} = 1$, log-linearizing the capital accumulation equation (CS 15) results in

$$\widetilde{k}_{t+1} = \delta(\widetilde{\nu}_t + \widetilde{i}_t) + (1-\delta)\widetilde{k}_t.$$

Therefore, multiplying (CS 21) by $(1-\delta)$, lagging it, and subtracting from the original equation results in

$$\begin{aligned}\widetilde{i}_t - (1-\delta)\widetilde{i}_{t-1} = \delta(\widetilde{\nu}_{t-1} + \widetilde{i}_{t-1}) + \frac{1}{\sigma\delta}\left[\beta\bar{r}^k\zeta\widetilde{u}_{t+1} - \widetilde{r}^p_t - (1-\delta)\left(\beta\bar{r}^k\zeta\widetilde{u}_t - \widetilde{r}^p_{t-1}\right)\right] + \\ + \beta\left(\widetilde{i}_{t+1} - (1-\delta)\widetilde{i}_t\right) - \beta\delta\left(\widetilde{\nu}_t + \widetilde{i}_t\right) + \varepsilon_t - (1-\delta)\varepsilon_{t-1},\end{aligned}$$

where $\widetilde{k}_t - (1-\delta)\widetilde{k}_{t-1}$ is replaced by $\delta(\widetilde{\nu}_{t-1} + \widetilde{i}_{t-1})$. Further simplification leads us to

$$\Delta\widetilde{i}_t = \beta\Delta\widetilde{i}_{t+1} + \frac{1}{\sigma\delta}\left[\beta\bar{r}^k\zeta\widetilde{u}_{t+1} - \widetilde{r}^p_t - (1-\delta)\left(\beta\bar{r}^k\zeta\widetilde{u}_t - \widetilde{r}^p_{t-1}\right)\right] + \tilde{\varepsilon}_{t-1}, \qquad \text{(CS 22)}$$

where $\tilde{\varepsilon}_{t-1} = -\beta\delta\widetilde{\nu}_t + \delta\widetilde{\nu}_{t-1} + \varepsilon_t - (1-\delta)\varepsilon_{t-1}$, or

$$\begin{aligned}\tilde{\varepsilon}_{t-1} = &-\left(\beta\delta - \frac{1-\beta\left(1-\delta\right)\rho}{\sigma\delta}\right)\widetilde{\nu}_t + \left[\delta - (1-\delta)\frac{1-\beta\left(1-\delta\right)\rho}{\sigma\delta}\right]\widetilde{\nu}_{t-1} \\ &- \frac{1}{\sigma\delta}\left[\beta\bar{r}^k\eta^u_{t+1|t} + \eta^\pi_{t+1|t} - (1-\delta)\left[\beta\bar{r}^k\eta^u_{t|t-1} + \eta^\pi_{t|t-1}\right]\right] - \beta\left[\eta^i_{t+1|t} - (1-\delta)\eta^i_{t|t-1}\right].\end{aligned}$$

We need to get rid of the $\tilde{\nu}$ in the error term. Therefore, lagging (CS 22), multiplying it by

ρ, and subtracting from (CS 22) results in

$$\Delta\widetilde{i}_t - \rho\Delta\widetilde{i}_{t-1} = \beta\left(\Delta\widetilde{i}_{t+1} - \rho\Delta\widetilde{i}_t\right) + \frac{1}{\sigma\delta}\left[\beta\bar{r}^k\zeta\left(\widetilde{u}_{t+1} - \rho\widetilde{u}_t\right) - \left(\tilde{r}_t^p - \rho\tilde{r}_{t-1}^p\right)\right]$$
$$- \frac{(1-\delta)}{\sigma\delta}\left[\left(\beta\bar{r}^k\zeta\left(\widetilde{u}_t - \rho\widetilde{u}_{t-1}\right) - \left(\tilde{r}_{t-1}^p - \rho\tilde{r}_{t-2}^p\right)\right)\right] + \tilde{\varepsilon}_{t-1} - \rho\tilde{\varepsilon}_{t-2},$$

or

$$\Delta\widetilde{i}_t\left(1+\rho\beta\right) = \rho\Delta\widetilde{i}_{t-1} + \beta\Delta\widetilde{i}_{t+1} + \frac{\beta\bar{r}^k}{\sigma\delta}\zeta\widetilde{u}_{t+1} - \frac{1}{\sigma\delta}\tilde{r}_t^p - \frac{(1-\delta)\beta\bar{r}^k}{\sigma\delta}\zeta\widetilde{u}_t + \frac{1-\delta}{\sigma\delta}\tilde{r}_{t-1}^p$$
$$- \frac{\rho\beta\bar{r}^k}{\sigma\delta}\zeta\widetilde{u}_t + \frac{\rho}{\sigma\delta}\tilde{r}_{t-1}^p + \frac{\rho(1-\delta)\beta\bar{r}^k}{\sigma\delta}\zeta\widetilde{u}_{t-1} - \frac{\rho(1-\delta)}{\sigma\delta}\tilde{r}_{t-2}^p + \tilde{\varepsilon}_{t-1} - \rho\tilde{\varepsilon}_{t-2}.$$

The above equation can be rewritten as

$$\Delta\widetilde{i}_t = \frac{\rho}{1+\rho\beta}\Delta\widetilde{i}_{t-1} + \frac{\beta}{1+\rho\beta}\Delta\widetilde{i}_{t+1} + \frac{\beta\bar{r}^k}{\sigma\delta\left(1+\rho\beta\right)}\zeta\widetilde{u}_{t+1} - \frac{1}{\sigma\delta\left(1+\rho\beta\right)}\tilde{r}_t^p$$
$$- \frac{\left(1-\delta\right)\beta\bar{r}^k}{\sigma\delta\left(1+\rho\beta\right)}\zeta\widetilde{u}_t + \frac{1-\delta}{\sigma\delta\left(1+\rho\beta\right)}\tilde{r}_{t-1}^p - \frac{\rho\beta\bar{r}^k}{\sigma\delta\left(1+\rho\beta\right)}\zeta\widetilde{u}_t + \frac{\rho}{\sigma\delta\left(1+\rho\beta\right)}\tilde{r}_{t-1}^p$$
$$+ \frac{\rho(1-\delta)\beta\bar{r}^k}{\sigma\delta\left(1+\rho\beta\right)}\zeta\widetilde{u}_{t-1} - \frac{\rho(1-\delta)}{\sigma\delta\left(1+\rho\beta\right)}\tilde{r}_{t-2}^p + \overline{\varepsilon}_{t-1}, \tag{CS 23}$$

where $\overline{\varepsilon}_{t-1}\left(1+\rho\beta\right) := \left(\tilde{\varepsilon}_{t-1} - \rho\tilde{\varepsilon}_{t-2}\right)$ is

$$-\beta\delta\widetilde{\nu}_t + \delta\widetilde{\nu}_{t-1} - \frac{1}{\sigma\delta}\left[\beta\bar{r}^k\eta_{t+1|t}^u + \eta_{t+1|t}^\pi\right] - \beta\eta_{t+1|t}^i + \frac{1-\beta\left(1-\delta\right)\rho}{\sigma\delta}\widetilde{\nu}_t$$
$$- \left(1-\delta\right)\left[-\frac{1}{\sigma\delta}\left[\beta\bar{r}^k\eta_{t|t-1}^u + \eta_{t|t-1}^\pi\right] - \beta\eta_{t|t-1}^i + \frac{1-\beta\left(1-\delta\right)\rho}{\sigma\delta}\widetilde{\nu}_{t-1}\right]$$
$$+ \rho\beta\delta\widetilde{\nu}_{t-1} - \rho\delta\widetilde{\nu}_{t-2} + \frac{\rho}{\sigma\delta}\left[\beta\bar{r}^k\eta_{t|t-1}^u + \eta_{t|t-1}^\pi\right] + \rho\beta\eta_{t|t-1}^i - \rho\frac{1-\beta\left(1-\delta\right)\rho}{\sigma\delta}\widetilde{\nu}_{t-1}$$
$$+ \rho(1-\delta)\left[-\frac{1}{\sigma\delta}\left[\beta\bar{r}^k\eta_{t-1|t-2}^u + \eta_{t-1|t-2}^\pi\right] + \rho\beta\eta_{t-1|t-2}^i - \rho\frac{1-\beta\left(1-\delta\right)\rho}{\sigma\delta}\widetilde{\nu}_{t-2}\right].$$

Further simplifying the above equation becomes

$$
\begin{aligned}
\overline{\varepsilon}_{t-1}(1+\rho\beta) = & -\beta\delta\varepsilon_t^v + \delta\varepsilon_{t-1}^v - \frac{1}{\sigma\delta}\left[\beta\bar{r}^k\eta_{t+1|t}^u + \eta_{t+1|t}^\pi\right] - \beta\eta_{t+1|t}^i + \frac{1-\beta(1-\delta)\rho}{\sigma\delta}\varepsilon_t^v \\
& -(1-\delta)\left[-\frac{1}{\sigma\delta}\left[\beta\bar{r}^k\eta_{t|t-1}^u + \eta_{t|t-1}^\pi\right] - \beta\eta_{t|t-1}^i + \frac{1-\beta(1-\delta)\rho}{\sigma\delta}\varepsilon_{t-1}^v\right] \\
& + \frac{\rho}{\sigma\delta}\left[\beta\bar{r}^k\eta_{t|t-1}^u + \eta_{t|t-1}^\pi\right] + \rho\beta\eta_{t|t-1}^i \\
& + \rho(1-\delta)\left[-\frac{1}{\sigma\delta}\left[\beta\bar{r}^k\eta_{t-1|t-2}^u + \eta_{t-1|t-2}^\pi\right] + \rho\beta\eta_{t-1|t-2}^i\right].
\end{aligned}
\tag{CS 24}
$$

Then, grouping some terms from above we get

$$
\begin{aligned}
\Delta\widetilde{i}_t = & \frac{\rho}{1+\rho\beta}\Delta\widetilde{i}_{t-1} + \frac{\beta}{1+\rho\beta}\Delta\widetilde{i}_{t+1} + \frac{\beta\bar{r}^k}{\sigma\delta(1+\rho\beta)}\zeta\widetilde{u}_{t+1} - \frac{1}{\sigma\delta(1+\rho\beta)}\tilde{r}_t^p \\
& - \frac{\beta\bar{r}^k(1-\delta+\rho)}{\sigma\delta(1+\rho\beta)}\zeta\widetilde{u}_t + \frac{1-\delta+\rho}{\sigma\delta(1+\rho\beta)}\tilde{r}_{t-1}^p + \frac{\rho(1-\delta)\beta\bar{r}^k}{\sigma\delta(1+\rho\beta)}\zeta\widetilde{u}_{t-1} \\
& - \frac{\rho(1-\delta)}{\sigma\delta(1+\rho\beta)}\tilde{r}_{t-2}^p + \overline{\varepsilon}_{t-1}.
\end{aligned}
\tag{CS 25}
$$

It can be gauged from looking at the equation (CS 24) that we need to use at least the second lag of endogenous variables as instruments in order to ensure exogeneity.

Figure S.5 shows that confidence sets are large suggesting weak identification of the structural parameters also in the capital adjustment cost model.

F On Cross-equation Restrictions and Identification

In this Subsection, we use a simple example to demonstrate how cross-equation restrictions from a system method can achieve identification of a model that is not identified using a single-equation GMM approach at the cost of losing robustness to misspecification.

Recall that GMM estimates the single equation (2), where we have also assumed that β and δ are known. For the purpose of this discussion it suffices to simplify the exposition to the case of a single unknown parameter. Hence, assume $\rho = 0$ and κ is known, so there is only one unknown parameter, ζ, and the model in equation (2) can be written as

$$
E_t y_{t+2} = \zeta E_t x_{t+1} + \widetilde{\nu}_t \tag{CS 26}
$$

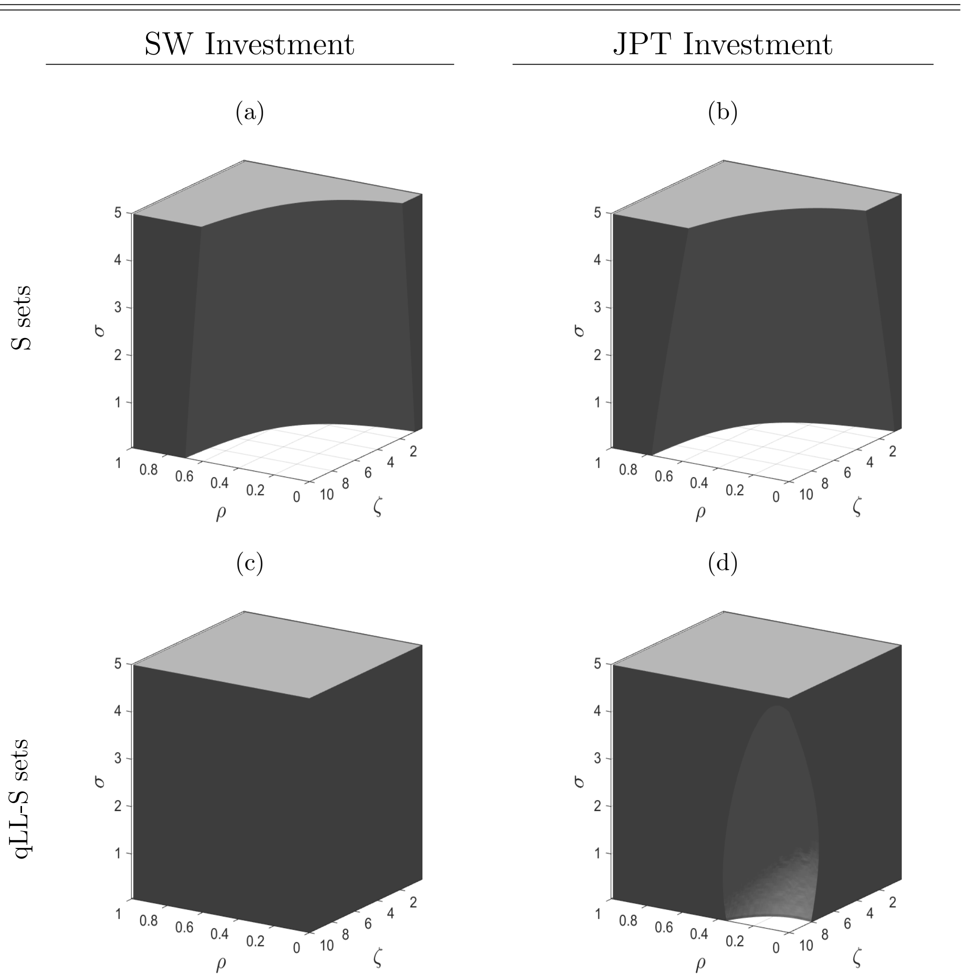

Figure S.5: 90% S and qLL-S confidence sets for $\theta = (\rho, \sigma, \zeta)$ derived from the investment Euler equation model (CS 25). Instruments: constant, Δi_{t-2}, r^p_{t-3}, u_{t-2}. The investment proxies are Fixed Private Investment (left column) and the sum of Gross Private Domestic Investment and Personal Consumption Expenditure on Durable Goods (right column). Newey and West (1987) HAC. Period: 1967Q1-2019Q4.

where $y_{t+2} := \kappa \left(\Delta \widetilde{i}_t - (\beta + \phi_q) \Delta \widetilde{i}_{t+1} + \beta \phi_q \Delta \widetilde{i}_{t+2} \right) + \widetilde{r}_t^p$ and $x_t := \phi_k \widetilde{u}_t$, with $\phi_k = 1 - (1 - \delta) \beta$.

Now, ζ is identified in (CS 26) if and only if $var\,(E_t x_{t+1}) > 0$. But in a limited-information setting, we do not observe $E_t x_{t+1}$, so we have to instrument for it using predetermined variables Z_t that belong to the information set at time $t-1$. Specifically, the corresponding single-equation GMM regression for (CS 26) is

$$y_{t+2} = \zeta x_{t+1} + \underbrace{[\widetilde{\nu}_t + y_{t+2} - E_t y_{t+2} - \zeta (x_{t+1} - E_t x_{t+1})]}_{\xi_t}, \tag{CS 27}$$

with $E_{t-1} \xi_t = 0$, and any predetermined variable is a valid instrument for x_{t+1}. So, for the (single-equation) GMM approach to identify ζ it is necessary that $var\,(E_{t-1} x_{t+1}) > 0$.

It is possible to come up with examples where a system method will identify ζ while the single-equation GMM approach will not. Suppose

$$x_t = \omega_t + \theta \omega_{t-1}, \quad |\theta| < 1, \tag{CS 28}$$

(an invertible first-order moving average process) and ω_t (the structural shock driving capacity utilization $x_t = \phi_k \widetilde{u}_t$) is orthogonal to the investment-specific technology shock $\widetilde{\nu}_t$. Equation (CS 28) implies that $E_t x_{t+1} = \theta \omega_t$, while $E_{t-1} x_{t+1} = 0$. So, ζ can be identified if we use the additional equation (CS 28), but it is not identified from a single-equation approach that does not make enough assumptions to pin down $E_t x_{t+1}$.

In terms of implementation, because of the triangular nature of this simple example, i.e., because (CS 28) does not involve $\widetilde{\nu}_t$ or y_t, we can demonstrate how identification works as follows. First estimate (CS 28) to obtain ω_t and θ, next, compute $z_t := \theta \omega_t = E_t x_{t+1}$, and finally, estimate ζ from the regression

$$y_{t+2} = \zeta z_t + \underbrace{(\widetilde{\nu}_t + y_{t+2} - E_t y_{t+2})}_{e_t}. \tag{CS 29}$$

Unless $\theta = 0$ (rank condition), which would imply $z_t = 0$ for all t, the above regression identifies ζ, so a system analysis will produce bounded confidence sets, while the single-equation

GMM analysis based on (CS 26) that uses only predetermined variables to instrument for x_{t+1} will produce unbounded confidence sets.

Misspecification The increased precision of the system approach comes at the cost of lower robustness to misspecification. Suppose the true law of motion for x_t were given by

$$x_t = \gamma x_{t-1} + \omega_t, \tag{CS 30}$$

i.e., an AR(1) instead of an MA(1). Using (CS 28) instead of (CS 30), one would get an inconsistent estimate of $E_t x_{t+1}$, say $z_t^* = \theta^* \omega_t^*$, where θ^*, ω_t^* are the pseudo-true values of θ, ω_t in the MA(1) specification (CS 28), when the data is generated according to (CS 30). So, instead of using (CS 29), a misspecified system approach would be estimating ζ from the incorrect regression

$$y_{t+2} = \zeta z_t^* + \underbrace{[\zeta\,(z_t - z_t^*) + e_t]}_{e_t^*}, \qquad cov(z_t^*, e_t^*) \neq 0. \tag{CS 31}$$

This can be shown as follows. Suppose x_t follows (CS 30). Then, its first autocorrelation is γ. The pseudo true value of the coefficient θ in the MA(1) specification (CS 28) is obtained by solving the equation (see Hamilton, 1995, p. 49)

$$\gamma = \frac{\theta^*}{1 + \theta^{*2}}, \tag{CS 32}$$

and we can choose the (unique) invertible solution that satisfies $|\theta_i| < 1$. Given θ^*, the corresponding estimate of the shock in (CS 28), ω_t^*, can be solved from $\{x_t\}$ using the backward recursion

$$\omega_t^* = \sum_{j=0}^{\infty} (-\theta^*)^j \, x_{t-j}, \tag{CS 33}$$

while the true structural shock in (CS 30) is simply

$$\omega_t = x_t - \gamma x_{t-1}.$$

So, we have

$$
\begin{aligned}
cov\left(z_t^*,\left(z_t-z_t^*\right)\right) &= cov\left(z_t^*, z_t\right)+\theta^{*2} var\left(\omega_t^*\right) \\
&= \frac{\sigma_\omega^2 \gamma \theta^*}{1-\gamma^2} \frac{1}{1-\left(\gamma \theta^*\right)^2}+\theta^{*2} \frac{\left(1-\gamma \theta^*\right) \sigma_\omega^2}{\left(1+\gamma \theta^*\right)\left(1-\gamma^2\right)\left(1-\theta^{*2}\right)} \\
&= \sigma_\omega^2 \theta^* \frac{\gamma\left(1-\theta^{*2}\right)+\theta^*\left(1-\gamma \theta^*\right)^2}{\left(1-\left(\gamma \theta^*\right)^2\right)\left(1-\gamma^2\right)\left(1-\theta^{*2}\right)},
\end{aligned} \tag{CS 34}
$$

because

$$
\begin{aligned}
cov\left(z_t^*, z_t\right) &= cov\left(\theta^* \sum_{j=0}^{\infty}\left(-\theta^*\right)^j x_{t-j}, \gamma x_t\right) \\
&= \frac{\sigma_\omega^2 \theta^* \gamma}{1-\gamma^2} \sum_{j=0}^{\infty}\left(-\theta^* \gamma\right)^j=\frac{\sigma_\omega^2 \gamma \theta^*}{1-\gamma^2} \frac{1}{1-\left(\gamma \theta^*\right)^2}
\end{aligned}
$$

and

$$
x_t=\omega_t^*\left(1+\theta^* L\right)=\omega_t(1-\gamma L)^{-1},
$$

so ω_t^* is an AR(2) process, $\omega_t^*\left(1-a_1 L-a_2 L^2\right)=\omega_t$, with $a_1=\gamma-\theta^*$, and $a_2=\gamma\theta^*$, and therefore, its variance is given by (see Hamilton, 1995, p. 58)

$$
\begin{aligned}
var\left(\omega_t^*\right) &= \frac{1-a_2}{1+a_2} \frac{\sigma_\omega^2}{\left(1-a_2\right)^2-a_1^2} \\
&= \frac{1-\gamma \theta^*}{1+\gamma \theta^*} \frac{\sigma_\omega^2}{\left(1-\gamma \theta^*\right)^2-\left(\gamma-\theta^*\right)^2} \\
&= \frac{\left(1-\gamma \theta^*\right) \sigma_\omega^2}{\left(1+\gamma \theta^*\right)\left(1-\gamma^2\right)\left(1-\theta^{*2}\right)} .
\end{aligned}
$$

Substituting for γ using θ^* from (CS 32) in (CS 34), we get

$$
cov\left(z_t^*,\left(z_t-z_t^*\right)\right)=\frac{\sigma_\omega^2 \theta^{*2}\left(2-\theta^{*4}\right)\left(\theta^{*2}+1\right)^2}{\left(1-\theta^{*2}\right)\left(1-\theta^*+\theta^{*2}\right)\left(1+\theta^*+\theta^{*2}\right)\left(1+2 \theta^{*2}\right)} \neq 0 .
$$

It follows, therefore, that (CS 31) suffers from omitted variable bias because z_t^* correlates with e_t^*. Thus, the system estimate of ζ will be biased.

A DSGE model allows us to use cross-equation restrictions to determine $E_t x_{t+1}$ under rational expectations, see Footnote 4. In this present simple example, one may think that we are not actually using any cross-equation restrictions because $E_t x_{t+1} = \theta \omega_t$ does not involve the structural parameter ζ of the original target equation (CS 26). However, in more general (non-triangular) settings where x_t is allowed to be simultaneously determined with y_t, $E_t x_{t+1}$ will depend also on ζ, and system estimation will indeed impose cross-equation restrictions.

G Prior-posterior distributions using JPT's model and SW's dataset

Investment adjustment cost (κ)	Elasticity capital utilization cost (ζ)	AR(1) coefficient investment shock (ρ)
(a)	(b)	(c)

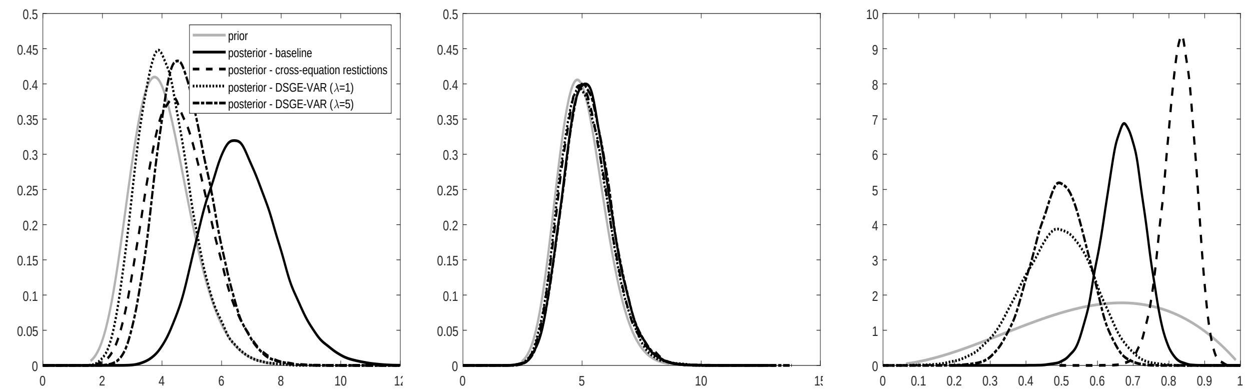

Figure S.6: Prior-posterior plots using JPT's model and SW's dataset. All estimations are done using Dynare. The posterior distributions are based on 500,000 draws, with the first 50% draws discarded as burn-in draws. The average acceptance rate is around 25-30%.

Investment adjustment cost (κ)	Inverse of elasticity capital utilization cost ($\psi = \frac{1}{\zeta}$)	AR(1) coefficient investment shock (ρ)

(a) (b) (c)

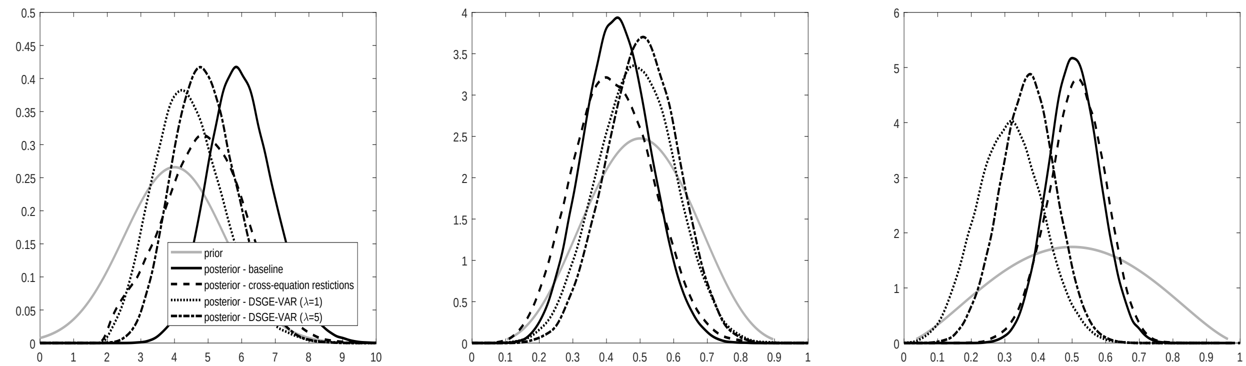

Figure S.7: Prior-posterior plots using SW's model and JPT's dataset. All estimations are done using Dynare. The posterior distributions are based on 500,000 draws, with the first 50% draws discarded as burn-in draws. The average acceptance rate is around 25-30%.

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