Supplementary Material to "How to Identify and Forecast Bull and Bear Markets?"

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This document contains supplementary material for the paper "How to Identify and Forecast Bull and Bear Markets?". The tables and figures belonging to a section are directly included after that section. Plain references refer to equations, figures and tables in the original paper. References to equations, tables and figures in the appendix are preceded by a capital letter.

A Rules for identifying bull and bear markets

A.1 Lunde and Timmermann (2004)

The identification rules in Lunde and Timmermann (2004) based on an (adjusted) index P_t can be summarized as follows:

1. Suppose the last observed extreme value before period t was a peak with index value $P^{\max}$.

   (a) If the index value P_t exceeds $P^{\max}$, the maximum is updated, $P^{\max} = P_t$.

   (b) If the index value P_t is more than a fraction λ_2 below $P^{\max}$, a trough has been found, and $P^{\min} = P_t$.

   (c) If neither of these conditions is satisfied, no update takes place.

   The agent now moves to $t + 1$.

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2. Suppose the last observed extreme value before period t was a trough with index value $P^{\min}$.

   (a) If the index value P_t is below $P^{\min}$, the minimum is updated, $P^{\min} = P_t$.

   (b) If the index exceeds $P^{\min}$ by more than a fraction λ_1, a peak has been found, and $P^{\max} = P_t$.

   (c) If neither of these conditions in satisfied, no update takes place.

   The agent now moves to $t + 1$.

To commence the procedure we have to determine whether the market is initially in a bull or bear state. For this purpose we count the number of times the maximum and minimum of the index have to be adjusted since the first observation. If the maximum has to be adjusted three times first, the market starts in a bull state, and in bear state otherwise.

A.2 Pagan and Sossounov (2003)

The algorithm of Pagan and Sossounov (2003) consists of five steps:

1. Identify all local maxima and minima in a price series. A local maximum (minimum) is higher (lower) than all prices in the past and future τ_{window} periods.

2. Construct an alternating sequence of peaks and troughs by selecting the highest maxima and lowest minima, in case two (or more) extrema of the same type occur consecutively.

3. Censor peaks and troughs in the first and last τ_{censor} periods.

4. Eliminate cycles of bull and bear markets that last less than τ_{cycle} periods.

5. Eliminate bull market or bear markets that lasts less than τ_{phase} periods, unless the absolute price change exceeds a fraction ζ.

We mostly follow PS for the values of these parameters, adjusted for the weekly frequency of our data. We set $\tau_{\text{window}} = 32$, $\tau_{\text{cycle}} = 70$, $\tau_{\text{phase}} = 16$ and $\zeta = 0.20$ (see also PS, Appendix B). We censor switches in the first and last 13 weeks, opposite to the 26 weeks taken by PS. Censoring for 26 weeks would mean that only after half a year an investor can be sure whether a bear or a bull market prevails, which we consider a very long delay. Since we will use this information for making forecasts, we use a shorter period of 13 weeks to establish the initial and the ultimate state of the market.

B Predictor variables

We consider several macroeconomic and financial variables that may predict bull and bear markets. For macroeconomic variables we calculate the inflation rate as the monthly change in log CPI, the growth rate of industrial production as the yearly change in the log industrial production, and the annual change in the unemployment rate from ALFRED database at the Federal Reserve Bank of St. Louis[1]. These data series contain vintage data, meaning that we use the unrevised values. For inflation, the vintage series starts only on July 21, 1972. We substitute the inflation rates (based on the CPIAUCSL series) from the FRED database of the St. Louis FED[2] for the missing period. These three series have a monthly frequency.

We also gather five financial variables. From the FRED database, we obtain the 3-month T-bill rate (WTB3MS) and calculate the term spread (10-year yield, WGS10YR, minus the T-bill rate) and the credit spread (Moody's BAA minus AAA corporate bond yield, seasonally adjusted). We calculate the dividend-to-price ratio based on the dividend series for the S&P500 that is available on Robert Shiller's homepage[3]. The series consists of monthly observations of the moving total dividends over the past twelve months. To calculate the D/P-ratio at time t (in weeks), we divide the dividends over the last 12 months prior to the month corresponding with time t by the current price level. We construct a variance series σ_t^2 for the S&P500 by the RiskMetrics method,

$$\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1-\lambda) r_{t-1}^2, \tag{B.1}$$

where λ is the decay factor. Because JP Morgan and Reuters (1996); Mina and Xiao (2001) advise $\lambda = 0.94$ for monthly data and $\lambda = 0.97$ for daily data, we use $\lambda = 0.95$ as we work with weekly data. We use the first 51 weeks to construct the variance estimate for that start of our sample (December 29, 1961). We use the resulting volatility as the predictor variable.

We test whether the variables are stationary by an Adjusted Dickey-Fuller test. The results in Table B.1 indicate that a unit root is rejected for all series except the T-bill rate and the D/P-ratio. We transform these two variables by taking the difference with respect to the yearly moving average as in Campbell (1991); Rapach et al. (2005). For volatility we do not conduct an ADF test, because this series is I(1) by construction. We standardize each variable using the means and standard deviation. As a consequence,

[1] See `http://alfred.stlouisfed.org/`.

[2] See `https://research.stlouisfed.org/fred2/`.

[3] See `http://www.econ.yale.edu/~shiller/data.htm` for more information.

all logit-coefficients relate to a one-standard deviation change, and the impact of the different variables can directly be compared. In the selection procedure, we exclude a variable-transition combination when a coefficient exceeds 3 in absolute value.

[Table B.1 about here.]

B.1 Bootstraps for in-sample testing

We follow the bootstrap procedure used by Rapach and Wohar (2006); Kilian (1999) and Kothari and Shanken (1997) to test for equality of in-sample performance in Section 2. In this procedure a simulated series of the predictor variables is constructed, too. To ensure that they are similar to the original series, we construct ARMA models for the predictors. The residuals from the models are the basis for the simulated series. We use the series of simulated residuals to construct a simulated series of each variable. Next we standardize each series. We use the data from November 1961 (monthly series) and from December 29, 1961 (weekly series) to initialize each series.

We determine the specification of the ARMA-models by checking whether the correlograms of the residuals are close to white noise. We also consider information criteria to select from the different specifications. The results in Table B.2 show that the models for the three macro variables contain two AR-lags and one MA-lag. Because the growth in industrial production and the change in unemployment are yearly changes, the MA(12) lag is also significant. The models for the weekly series are simpler. Their dynamics are sufficiently captured by an ARMA(1,1) model. For the D/P-ratio the MA(1)-lag is insignificant, and we therefor omit it from the model. We construct simulated volatility series based on the simulated return series.

[Table B.2 about here.]

Table B.1: Characteristics of Predictor Variables

Variable	Frequency	ADF	p-value	Transf.	Mean (in %)	Std. Dev. (in %)
Inflation	Monthly	-5.01	$<.001$	No	0.327	0.33
Ind. Prod. Growth	Monthly	-5.76	$<.001$	No	2.572	4.79
Unemployment Change	Monthly	-4.02	0.001	No	0.016	1.07
T-bill	Weekly	-2.03	0.275	Yes	-0.023	1.04
Term Spread	Weekly	-4.40	$<.001$	No	1.529	1.28
Credit Spread	Weekly	-4.02	$<.001$	No	1.029	0.46
D/P ratio	Weekly	-1.56	0.504	Yes	-0.009	0.33
RM Volatility	Weekly	–	–	No	2.046	0.79

This table shows the set of predictor variables. We conduct an Augmented Dickey-Fuller (ADF) test on each variable except volatility. We report the ADF test statistic and the p-value for the null hypothesis of the presence of a unit root. If this hypothesis is not rejected, we transform the series by taking the difference with respect to the average over the preceding year. The last two columns give the mean and standard deviation of the (transformed) variables in %. The monthly series run from November 1960 to November 2013 (625 observations); the (transformed) weekly series from December 29, 1961 to December 27, 2013 (2714 observations).

Table B.2: ARMA Models for Predictor Variables

variable	c		AR(1)		AR(2)		MA(1)		MA(12)	
Inflation	0.334	(0.092)	1.328	(0.052)	−0.339	(0.048)	−0.901	(0.030)		
Ind. Prod. Growth	2.285	(0.397)	1.638	(0.043)	−0.658	(0.043)	−0.318	(0.046)	−0.465	(0.036)
Unemployment Change	0.043	(0.059)	1.374	(0.048)	−0.382	(0.047)	−0.321	(0.029)	−0.654	(0.027)
T-bill	−0.027	(0.168)	0.972	(0.005)			0.260	(0.019)		
Term Spread	1.571	(0.305)	0.987	(0.003)			0.228	(0.019)		
Credit Spread	1.033	(0.124)	0.990	(0.003)			0.256	(0.019)		
D/P ratio	−0.008	(0.055)	0.974	(0.004)						

This table shows the specification of ARMA models for the different predictor variables. The specification is determined by inspection of the correlograms of the residuals and information criteria. We report coefficient estimates with standard errors in parentheses.

C Additional Results

C.1 Three-state Markov-switching models

The RS3C model extends the RS2C model. It contains a third regime in which excess returns follow a normal distribution. When the transition probabilities are constant, the transition matrix contains six free parameters. If the probabilities are time-varying, we use a multinomial logit specification

$$\pi_{qst} \equiv \pi_{qs}(\boldsymbol{z}_{t-1}) \equiv \Pr[S_t = s | S_{t-1} = q, \boldsymbol{z}_{t-1}] = \frac{\mathrm{e}^{\boldsymbol{\beta}_{qs}^{m\prime} \boldsymbol{z}_{t-1}}}{\sum_{\varsigma \in \mathcal{S}} \mathrm{e}^{\boldsymbol{\beta}_{q\varsigma}^{m\prime} \boldsymbol{z}_{t-1}}}, \qquad s, q \in \mathcal{S}_m, \quad \text{(C.1)}$$

with $\exists s \in \mathcal{S} : \boldsymbol{\beta}_{qs} = \mathbf{0}$ to ensure identification. We have dropped the model-superscript m for notational convenience.

C.1.1 Estimation with time-varying transition probabilities

To estimate the parameters $\boldsymbol{\beta}_{qs}$, we extend the approach of Diebold et al. (1994), based on the EM-algorithm by Dempster et al. (1977). Diebold et al. (1994) consider estimation when the transition probabilities are linked via a standard (binomial) logit transformation. This extension maintains the attractive feature of the EM-algorithm that the expectation of the complete-data log likelihood can be split in terms related to only a subset of the parameter space. The transition part of the expectation of the likelihood function, which is only related to the parameters $\boldsymbol{\beta}_{qs}$, is given by

$$\ell(B) = \sum_{t=1}^{T} \sum_{s \in \mathcal{S}} \sum_{q \in \mathcal{S}} \xi_{qst} \log \pi_{qst}, \quad \text{(C.2)}$$

where $B = \{\boldsymbol{\beta}_{qs} : s, q \in \mathcal{S}\}$ is the set of all parameters $\boldsymbol{\beta}_{qs}$ and $\xi_{qst} \equiv \Pr[S_t = s | S_{t-1} = q, \Omega_T]$ is a smoothed state probability. These probabilities are based on the complete data set of returns and predictor variables Ω_T, and are calculated with the method of Kim (1994).

In the expectation step the set of smoothed state probabilities is determined. In the maximization step new parameters values are calculated that maximize the expected likelihood function. The first order conditions that apply to $\boldsymbol{\beta}_{qs}$ result from differentiating Equation (C.2)

$$\frac{\partial \ell}{\partial \boldsymbol{\beta}_{qs}} = \sum_{t=1}^{T} \sum_{\varsigma \in \mathcal{S}} \xi_{q\varsigma t} \frac{1}{\pi_{q\varsigma t}} \frac{\partial \pi_{q\varsigma t}}{\partial \boldsymbol{\beta}_{qs}}.$$

Based on Equation (C.1) we find

$$\frac{\partial \pi_{q\varsigma t}}{\partial \boldsymbol{\beta}_{qs}} = \begin{cases} \pi_{qst}(1-\pi_{qst})\boldsymbol{z}_{t-1} & \text{if } \varsigma = s \\ -\pi_{q\varsigma t}\pi_{qs}\boldsymbol{z}_{t-1} & \text{if } \varsigma \neq s \end{cases}.$$

Combining these two expressions yields the first order condition

$$\sum_{t=1}^{T} \left(\xi_{qst} - \xi_{q,t-1}\pi_{qst}\right) \boldsymbol{z}_{t-1} = \mathbf{0} \quad \forall q, s \in \mathcal{S} \tag{C.3}$$

where $\xi_{q,t} = \Pr[S_t = q|\Omega_T]$. For each departure state q the set of the first order conditions for the different $s \in \mathcal{S}$ comprise a system that determines the set $B_q = \{\boldsymbol{\beta}_{qs} : s \in \mathcal{S}\}$. Numerical techniques are used to find parameters $\boldsymbol{\beta}_{qs}$ that solve this system.

C.1.2 Marginal Effects with time-varying transition probabilities

Because the multinomial logit transformation is non-linear, the coefficients on the explanatory variables cannot be interpreted in a straightforward way. To solve this problem, we calculate the marginal effect of the change in one variable z_i, evaluated at specific values for all variables $\bar{\boldsymbol{z}}$. The marginal effect is given by the first derivative of (C.1) with respect to z_i:

$$\left.\frac{\partial \pi_{qs}(\boldsymbol{z})}{\partial z_i}\right|_{\boldsymbol{z}=\bar{\boldsymbol{z}}} = \pi_{qs}(\bar{\boldsymbol{z}}) \left(\beta_{qsi} - \sum_{\varsigma \in \mathcal{S}} \pi_{q\varsigma}(\bar{\boldsymbol{z}})\beta_{q\varsigma i}\right), \tag{C.4}$$

where β_{qsi} denotes the coefficient on z_i. It is easy to verify that the sum of this expression over the destination states s is equal to zero. Since the probabilities for the destination states should add up to one, a marginal increase in one probability should be accompanied by decreases in the other probabilities. When only two regimes are available, the above expression reduces to the familiar expression for marginal effects in logit models, $\pi_{qs}(\bar{\boldsymbol{z}})(1-\pi_{qs}(\bar{\boldsymbol{z}}))\beta_{qsi}$.

C.1.3 Results

The distribution parameters of the regimes in Table C.1 show a clear bull state that is similar to the RS2C bull state, and two bear states. One bear state is mild, with a slightly negative average return and a volatility of 2.50%. The other bear state is more extreme with prices that decrease on average by -0.87% per week and a volatility of 5.50%. The high standard errors indicate that the estimates for the strong bear regime are not very precise. The parameters for the RS3L model do not differ much from the RS3C model. The periods at which the strong bear state prevails are marked by purple areas in Figure C.1. They correspond with well-known crises and crashes in 1974, 1987, 2000, 2001 and 2008. The bull and mild bear state show an alternating pattern that looks more like the patterns in Figures 1a and 1b. However, we still see periods with volatile price increases marked as bear market, and periods with tranquil price decreases marked as bull market.

[Table C.1 about here.]

[Figure C.1 about here.]

We report the performance of the RS3 models in Table C.2 based on the full-sample and on the out-of-sample analysis, and compare it to the RS2 models. The identification results as well as the forecasting results show that the RS3 models lead to worse performing strategies than the RS2 models, though differences are small. In the case of identification, Sharpe ratios and realized utility are lower than for the corresponding RS2 models. As a consequent, fees are negative. An investor wants a compensation of -3.2 bps for a switch from RS2C to RS3C and -0.4 bps for a switch from RS2L to RS3L. Predictor variables improve performance, but fee of 1.0 bps shows that the effect is again small. The standard errors are high, indicating that these fees are not very precise, and can vary considerably from one sample to the next.

[Table C.2 about here.]

Including an additional regime does not improve forecasts. The realized average returns go down, while volatilities go up. As a consequences, both the Sharpe ratios and the realized utility are lower than for the RS2 models. As in the case of identification, fees are negative. An investor requires a compensation of -3.5 bps to switch from RS2C to RS3C, which is significant at the 5% level. A switch from RS2L to RS3L commands a compensation of -5.0 bps and is significant at the 10%-level. The forecasts in Figures C.2a and C.2b do not differ much from those in Figures 2e and 2f, so an explanation is not obvious. An inspection of the estimation results for the different windows in Appendix C.4 shows that the interpretation of the three regimes varies over time. A strong bear regime as in Table 1 is only identified when the subprime credit crisis is part of the estimation window. The use of predictor variables leads to slightly better forecasts, but the fee is not significant.

[Figure C.2 about here.]

Overall, we conclude that the addition of a third regime is not meaningful from an investment perspective. The third regime leads to a worse performance than the RS2 models, both for identification and for forecasting.

C.2 Constant and Time-Varying Transition Probabilities

Table C.3 reports the transition probabilities under the assumption that they are constant over time. Bull and bear markets are quite persistent with probabilities of around 0.95 or higher that the current state prevails for another week. Bull markets tend to be slightly more persistent than bear markets. Only the strong bear state is somewhat less persistent with a probability of continuation of 0.89. Since the beart state of the RS2H model captures crashes, it is not persistent. Both the mean and variance chains in the RS4C model are persistent. The unconditional regime probabilities in panel (b) indicate that bull markets appear approximately 2/3 of the time. The strong bear state of the RS3C model is unconditionally not very likely with a probability of only 0.009. The probability of the low-mean state in the RS4C model is also quite low at 0.07.

[Table C.3 about here.]

We analyse time-varying transition probabilities by linking them to predictor variables. We select the variables by the bottom-up procedure discussed in Section 3. The results in Table C.4 show that time-variation can be related to a few economic variables. The D/P ratio is selected for all models. A rise in the D/P-ratio increases the likelihood of a bull market in the next period in the LT and PS methods. This result does not comes as a surprise, since an increase in the D/P ratio can point at higher future expected returns. In the RS2L and RS3L models the D/P-ratio decreases the probability of a switch to a bull market, which is puzzling. It is similarly puzzling than an increase in the D/P-ratio strengthens the persistence of the strongly bearish regime in the RS3L model.

[Table C.4 about here.]

The selection of other variables does not show a clear pattern. For the LT method, a larger increase in the industrial production and an increase in the T-bill rate versus its yearly average lead to a larger probability of a switch to a bear market. For the PS method, higher inflation and an increase in the T-bill rate have the same effect, whereas a larger change in the unemployment rate or a higher credit spread make a switch to a bear market less likely. A higher term spread leads to an increase in the probability of a bear to bull switch, while an increase in the RiskMetrics volatility makes such a switch less likely. For the Markov-switching models, less variables are selected. In the RS2L model, an increase in the term spread makes continuation of a bull market more likely, whereas RiskMetrics volatility has the opposite effect. For the RS3L model, inflation and volatility are selected. Higher inflation makes a switch form the mild bear to the bull state more likely, while higher volatility increases the probability of the reverse switch.

We calculate the marginal effect of a one-standard deviation change in a predictor variable on a reference probability $\bar{\pi}$ to determine its impact. For reference probability we use the average forecast probability

$$\bar{\pi}_{qs} = \frac{\sum_{t=1}^{T} \Pr[S_{t+1} = s | S_t = q, \boldsymbol{z}_t] \Pr[S_t = q | \Omega_t]}{\sum_{t=1}^{T} \Pr[S_t = q | \Omega_t]}, \tag{C.5}$$

where Ω_t contains all observations up to time t. In this expression, each forecast probability $\Pr[S_{t+1} = s | S_t = q, \boldsymbol{z}_t]$ of a switch from state q to state s is weighted by the likelihood of an occurrence of state q at time t, $\Pr[S_t = q | \Omega_t]$. In the rule-based approaches, the weights are either zero or one. In the Markov-switching models the weights are the state probabilities.

The average forecast probabilities $\bar{\pi}_{qs}$ in the last row of Table C.4 confirm the strong persistence reported in Table C.3. For logit transformations, the marginal effect of variable z_i with coefficient β_i is given by $\bar{\pi}(1 - \bar{\pi})\beta_i$. For the multinomial logit transformation we derive the marginal effects in Equation (C.4). The impact of the D/P ratio is largest, with a marginal effect of around 0.015 for the LT method, 0.025 for the PS method and 0.078 for the RS2L model. In the RS3L model the effect is 0.03 on the probability of a mild-bear to bull switch, and -0.28 on a switch from the strong to mild bear to state. In this last case, the marginal effect may be poorly approximated. For the rule-based method, the impact of the other variables is small with a maximum of 0.01. In the Markov-switching models, the impact varies from 0.02 to 0.06.

We show the effect of time variation on the transition probabilities in the case of the full-sample identification in Figure C.3. These graphs show only minor differences compared to Figures 1c and C.1. The smoothed state probabilities are more volatile, but that does not lead to different bull and bear markets.

[Figure C.3 about here.]

Combining these results we conclude that all methods identify persistent bull and bear markets. The evidence for time-variation in the transition probabilities is limited.

However, if the sentiment of the stock market is a good predictor of other economic processes, it is not surprising that other economic variables fail to predict the stock market sentiment well. The D/P-ratio, which is closely related to expected returns in the stock market, is most consistently selected.

C.3 Model choice

To judge the quality of the different models, we report log likelihood values in Table C.5. For all models, the improvements by introducing predictor variables in the transition probabilities are significant by construction. Also, both the AIC and the BIC favor the models with time-variation in the transition probabilities, except for the RS3L model.

[Table C.5 about here.]

We can also evaluate the added value of an extra regime in the Markov-switching models. An additional regime leads to large improvements in the likelihood values, independent of transition probabilities being constant or time-varying. Both information criteria prefer three regimes over two regime specifications. When transition probabilities are constant, the model with two regimes is nested in the model with three regimes, and we could conduct a likelihood ratio test (the LR-statistic equals 105.50). However, due to presence of unidentified nuisance parameters under the null hypothesis of two regimes, the statistic does not follow a standard χ^2 distribution and simulations are needed. Because our interest is not in selecting the best statistical model, we do not conduct this test. Given the magnitude of the LR-statistic and the values of the information criteria, the evidence so far supports a model with three regimes. The criteria indicate that the RS2H model is the worst specification.

C.4 Additional Results on Forecasting

In our forecasting exercise we reestimate the parameters of the different models every 52 weeks. Here we show how the different parameters evolve. Besides aiding our understanding of the forecasts, it also shows how robust the parameters and characteristics are when more information becomes available.

Figure C.4 shows the evolution of the means and volatilities of the different regimes. Generally, these are stable. As we concluded in the full-sample analysis, the difference between the mean for the bull and for the bear regime is more extreme for the rules-based approaches, while the difference between the volatilities for these two regimes is more extreme for the RS2 models. The means in the RS4C model fluctuate a bit more, indicating that their estimates are less precise.

[Figure C.4 about here.]

The evolution of the means and volatilities of the regimes in the RS3 models in Figures C.4e and C.4f shows the influence of the credit crisis. When data until May 2008 is used, a clear bull regime (positive mean, low volatility), a clear bear regime (negative mean, high volatility) and an intermediate regime (mean close to zero, volatility in between) are identified. The characteristics of the bear state match quite well with those of the bear state in the RS2 models. After 2008, a bull, a mild bear and a strong bear state show up. The strong bear state reflects the exceptional behavior of the US stock market during the credit crisis.

The evolution of the transition probabilities when assumed constant within the estimation window are in Figure C.5. The methods with two states produce transition probabilities that are also quite stable over time. Persistence is high for both regimes. The probability of remaining in a bull state never falls below 0.95. In the rules-based approaches, the same applies to the bear regime. In the RS2C model, a bear market seems slightly less persistent, but the probability of continuation almost always exceeds 0.90.

[Figure C.5 about here.]

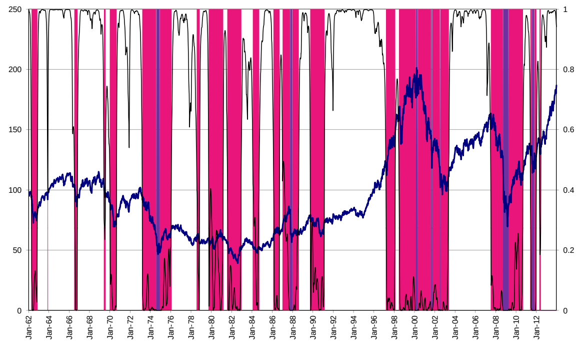
The transition probabilities in the RS3C model vary a bit more than in the two-state models, in particular the probabilities of a switch to a different regime than the departure regime. For example, the probability of a switch from the strong bear to the mild bear state ranges from 0.06 to 0.15. The probabilities that a specific regime continues are high (around 0.90 or more) and quite stable. The transition probabilities of the mean and variance chains of the RS4C model also show persistence. For some subsamples, the bull state in the mean chain is an absorbing state, which indicates that this method cannot always identify switches between periods with a high and a low mean.

The parameter dynamics of the models with time-varying transition probabilities in Figure C.6 indicate whether the parameters are stable, but also whether the variable selection is stable. In the two-state methods, we see some variable-combinations that show up for each estimation window with small variations in the estimated parameters, in particular the D/P ratio, the term spread and RiskMetrics volatility. For the other variables in the rule-based methods we see quite some variation, in particular when the bull state is the departure state. The variables that are selected for the RS2L model do not vary that much. For the RS3L model we see more variation, but a clear pattern is absent. Only the credit spread for the bull to bull transition and the D/P-ratio for the strong bear to mild bear transition are selected with some consistency. Here the selection also seems less reliable.

[Figure C.6 about here.]

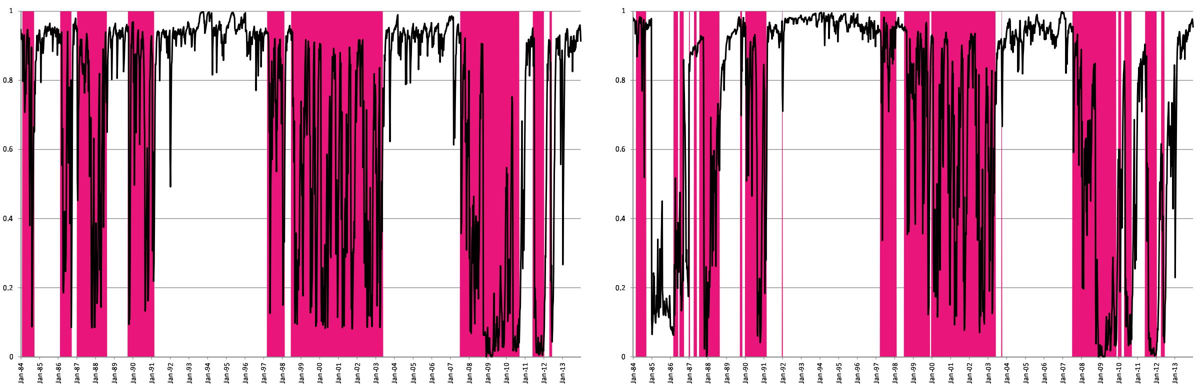
[Figure C.6 (continued) about here.]

Figure C.1: Identification based on the MS3C model



This figure shows the identification of the different regimes in the MS3C model. The thick blue line plots the excess stock market index (left y-axis). Mild bear (strong bear) markets are indicated with pink (purple) areas. A particular regime prevails when the smoothed probability for that regime exceeds 0.5.

Figure C.2: Forecasts of the MS3 models

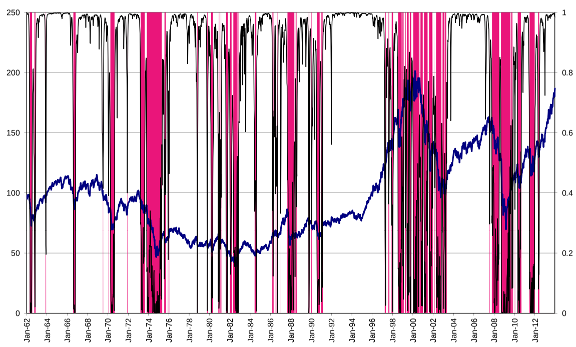


(a) MS3C

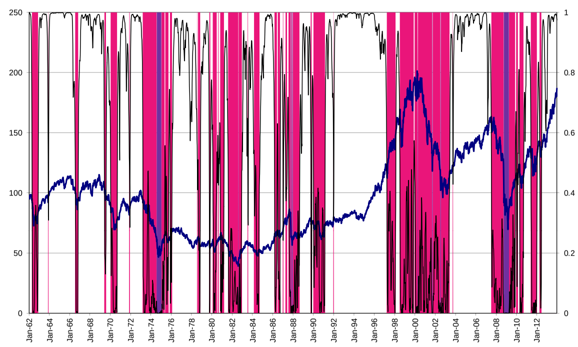
(b) MS3L

See Figure 2 for a general description. We calculate the probability of a bull market as the sum of the forecasts for regimes with positive means.

Figure C.3: Identification of Bull and Bear Markets with Time-Varying Transition Probabilities



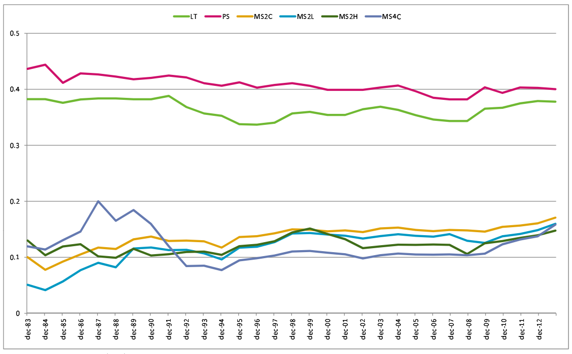
(a) RS2L



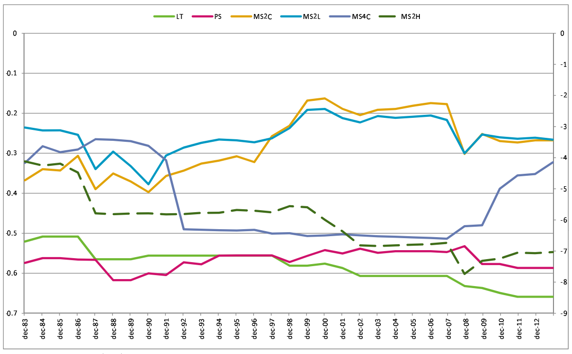
(b) RS3L

See Figure 1.

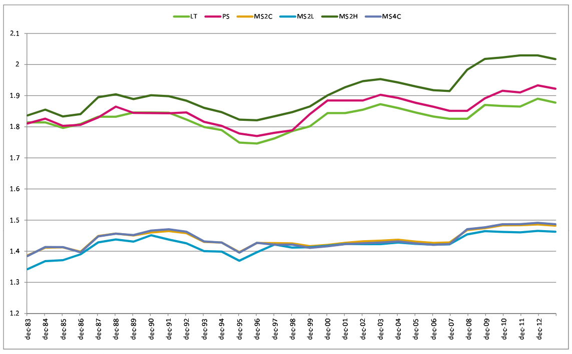
Figure C.4: Evolution of means and volatilities per regime



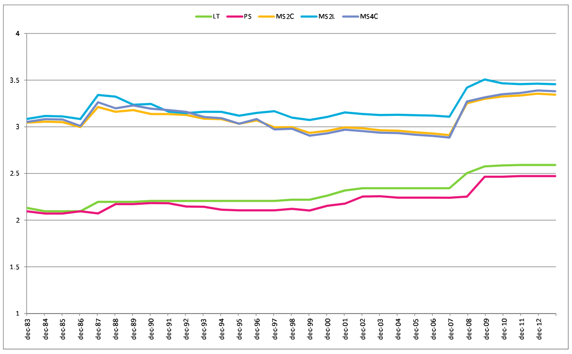
(a) bull mean, two-state methods



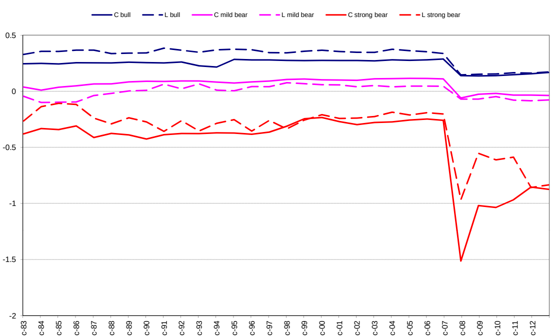
(b) bear mean, two-state methods



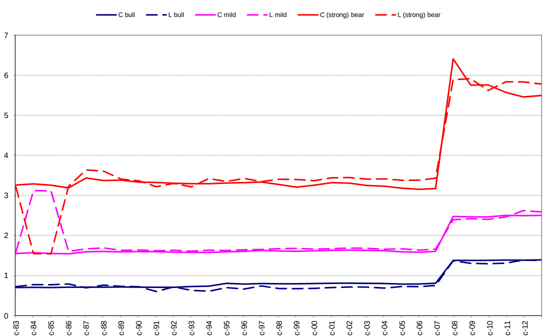
(c) bull volatility, two-state methods



(d) bear volatility, two-state methods



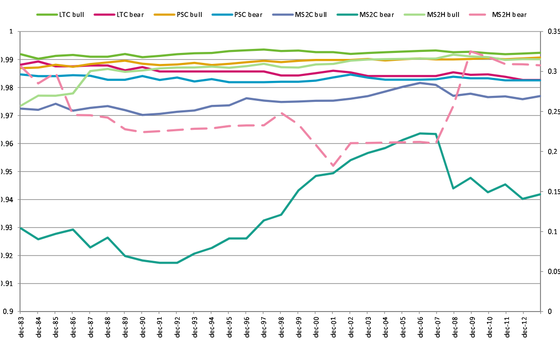
(e) means, MS3-methods



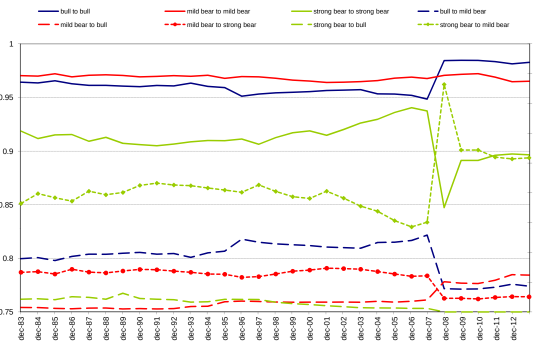
(f) volatility, MS3-methods

This figure shows the evolution of the means and volatilities when estimated with an expanding window (end date on the x-axis). The first window comprises the period January 5, 1962 – December 30, 1983 (1148 observations), and is continuously expanded with 52 weeks until we reach the end of the sample (December 27, 2013). In the case of LT and PS, we first identify the sequences of bull and bear markets in each estimation window, and then estimate means and volatilities per regime. The means and volatilities of the regimes in the Markov-switching models follow directly from the estimation. The dashed line for the mean of the bear regime in the RS2H-model in panel (b) corresponds with the secondary y-axis.

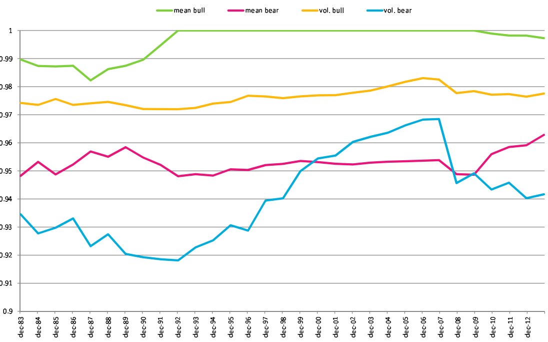
Figure C.5: Evolution of constant transition probabilities



(a) 2-state methods



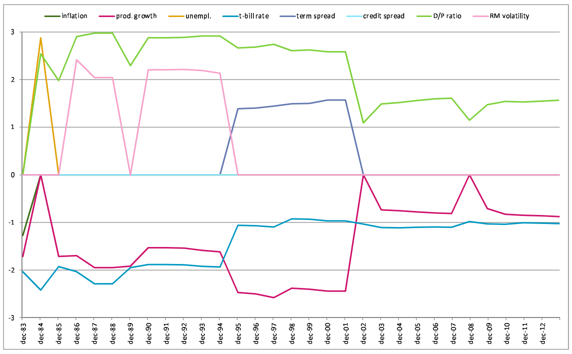
(b) MS3C



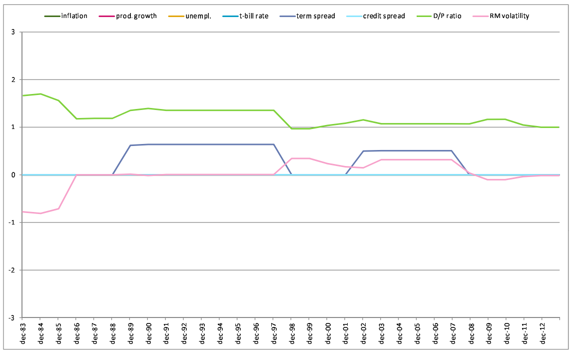
(c) MS4C

This figure shows the evolution of the transition probabilities when estimated with an expanding window (end date on the x-axis). The first window comprises the period January 5, 1962 – December 30, 1983 (1148 observations), and is continuously expanded with 52 weeks until we reach the end of the sample (December 27, 2013). In the case of LT and PS, we first identify the sequences of bull and bear markets in each estimation window, and then estimate the transition probabilities. The transition probabilities in the Markov-switching models result directly from the estimation. We plot the probabilities of a bull-bull and a bear-bear switch resulting from the LT, PS and MS2C-methods in Panel (a). For the MS3C model in Panel (b) we indicate the transition in the legend above the subfigure. Dashed lines correspond with the secondary y-axis. We plot the probabilities that correspond with regime continuation in the chains of MS4C in Panel (c)

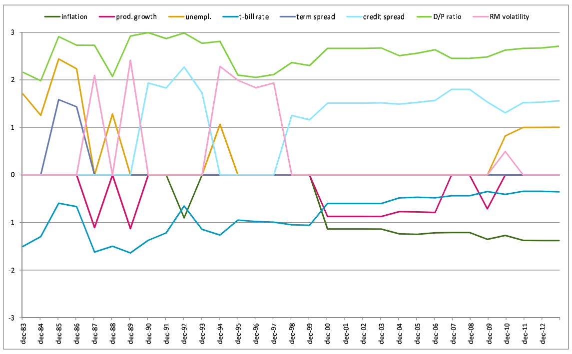
Figure C.6: Evolution of parameters in logit models



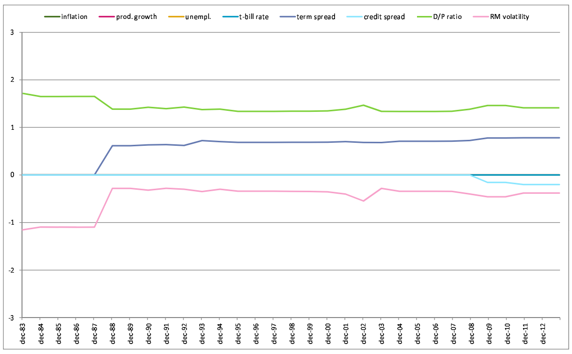
(a) LT, from bull to bull



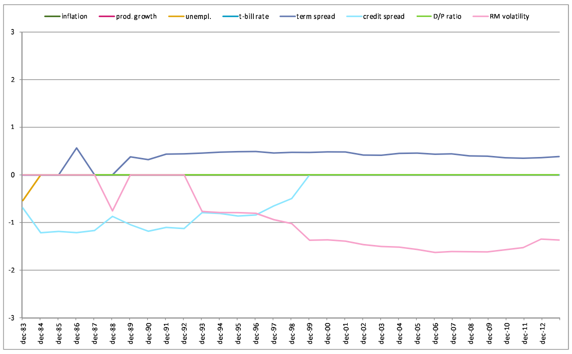
(b) LT, from bear to bull



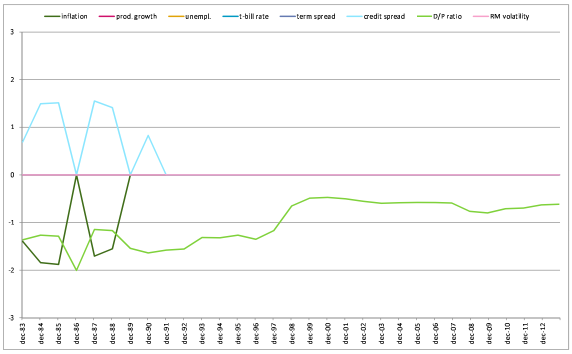
(c) PS, from bull to bull



(d) PS, from bear to bull



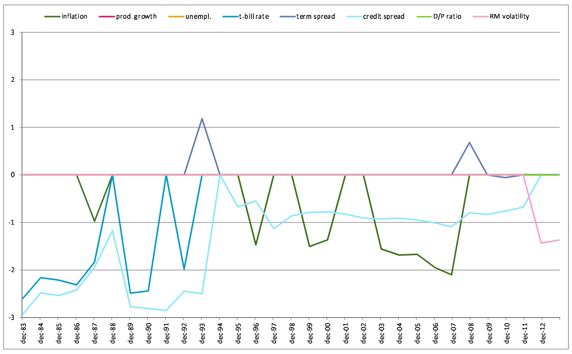
(e) RS2L, from bull to bull



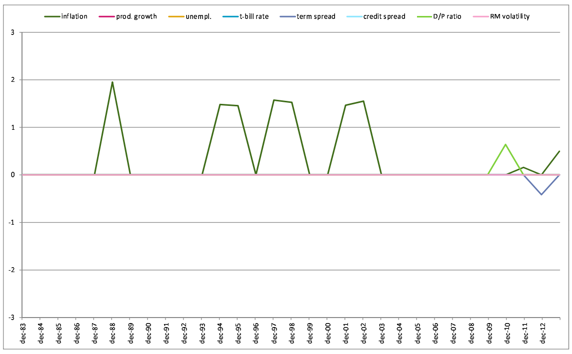
(f) RS2L, from bear to bull

This figure plots the evolution of the coefficients in the (multinomial) logit transitions for the predictor variables in Table B.1, when estimated with an expanding window (end date on the x-axis). The first window comprises the period January 5, 1962 – December 30, 1983 (1148 observations), and is continuously expanded with 52 weeks until we reach the end of the sample (December 27, 2013). The predictor variables have been standardized by subtracting their full-sample mean and dividing by their full-sample standard deviation. In the case of LT and PS, we first identify the sequences of bull and bear markets in each estimation window, and then estimate a dynamic logit model as in Equation (1), where the coefficients depend on the departure state. In the MS2L-model the logistic transformation is used to link the predictor variables to the transition probabilities. *Figure note continues on next page.*

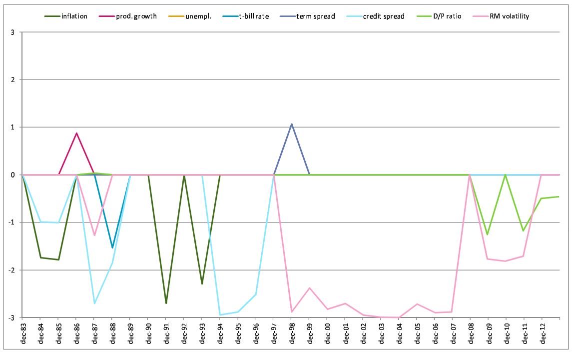
Figure C.6: Evolution of parameters in logit models – *continued*



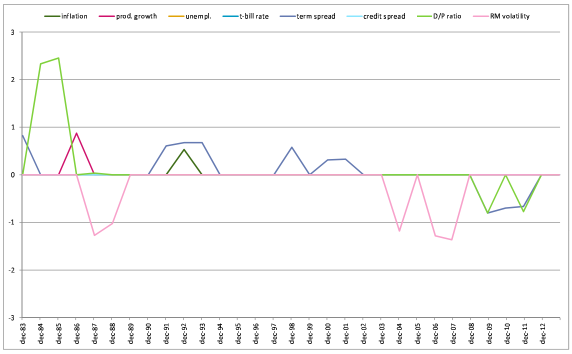
(g) MS3L, from bull to bull



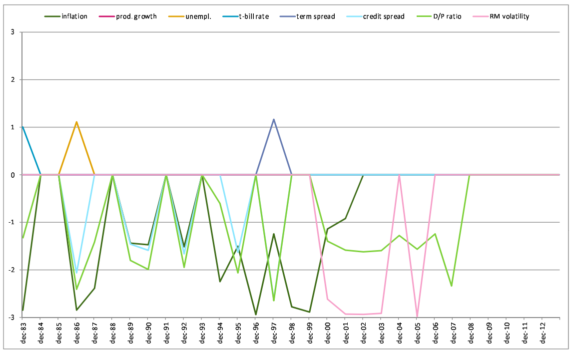
(h) MS3L, from bull to mild bear



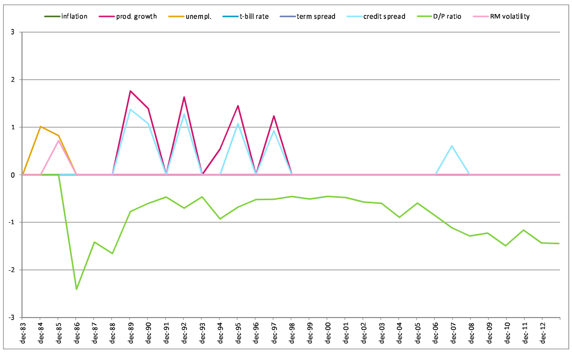
(i) MS3L, from mild bear to bull



(j) MS3L, from mild bear to mild bear



(k) MS3L, from strong bear to bull



(l) MS3L, from strong bear to mild bear

Continued from previous page. In the MS3L, the multinomial logistic transformation in Equation (C.1) is used, with all coefficients for a switch to the strong bear regime have been fixed at zero. The variable-transition combinations that subsequently produce the biggest increase in the likelihood function are included in the models. The procedure stops when the remaining variable-transition combinations fail to produce an increase in the likelihood function that is significant on the 10%-level.

Table C.1: Full Sample Moments of Regimes of the MS3 models

		MS3C	MS3L
bull	mean	0.17	0.17
		(0.04)	(0.04)
	vol.	1.39	1.38
		(0.03)	(0.04)
mild bear	mean	−0.04	−0.08
		(0.09)	(0.10)
	vol.	2.50	2.59
		(0.08)	(0.10)
strong bear	mean	−0.87	−0.83
		(0.60)	(0.75)
	vol.	5.49	5.78
		(0.38)	(0.49)

This table shows the mean and the volatility (in % per week) for the different regimes in the MS3 models. We report standard errors in parentheses, calculated by the outer product of the score to compute standard errors.

Table C.2: Performance Measures and Fees for the MS3 models

(a) Performance Measures for the full sample

	MS3C	MS3L
% of pos. weights	65	66
av. pos. weight	1.27	1.23
av. neg. weight	-0.19	-0.22
mean (in %)	0.16	0.17
volatility (in %)	1.59	1.50
Sharpe ratio (ann.)	0.75	0.80
utility ($\times 10,000$)	19.73	20.71

(b) Out-of-sample Performance Measures

	MS3C	MS3L
% of pos. weights	62	57
av. pos. weight	0.71	0.823
av. neg. weight	-0.31	-0.30
mean (in %)	0.009	0.032
volatility (in %)	1.44	1.61
Sharpe ratio (ann.)	0.04	0.14
utility ($\times 10,000$)	3.28	4.30

(c) Full-sample Fees (in bps)

	MS2L	MS3C	MS3L
MS2C	-1.8	-3.2	-2.3
	(0.6)	(10.7)	(11.4)
MS2L		-1.4	-0.4
		(11.0)	(11.4)
MS3C			1.0
			(2.6)

(d) Out-of-sample Fees (in bps)

	MS2L	MS3C	MS3L
MS2C	2.6	-3.5	-2.5
	[0.08]	[0.97]	[0.74]
MS2L		-6.1	-5.0
		[0.99]	[0.93]
MS3C			1.0
			[0.37]

For panels (a) and (c) see Table 2; for panels (b) and (d) see Table 3.

Table C.3: Constant Transition Probabilities

(a) Probability Estimates

from	to	LT	PS	MS2C	MS2H	MS3C	MS4C mean	MS4C variance
bull	bull	0.992	0.991	0.977	0.990	0.983	0.997	0.978
	(mild) bear	0.008	0.009	0.023	0.010	0.017	0.003	0.022
	strong bear					< 0.001		
(mild)	bull	0.017	0.017	0.058	0.692	0.025	0.037	0.058
bear	(mild) bear	0.983	0.983	0.942	0.308	0.965	0.963	0.942
	strong bear					0.01		
strong	bull					< 0.001		
bear	(mild) bear					0.103		
	strong bear					0.897		

(b) Unconditional Regime Probabilities

	LT	PS	MS2C	MS2H	MS3C	MS4C mean	MS4C variance
bull	0.695	0.655	0.716	0.986	0.750	0.930	0.722
(mild) bear	0.305	0.345	0.284	0.014	0.242	0.070	0.278
strong bear					0.009		

This table shows the transition probabilities between the different regimes in the different methods and the resulting unconditional regime probabilities. We assume that the probabilities are constant over time. In the approaches of LT and PS, we first apply their algorithms to identify the sequences of bull and bear markets. As a second step we estimate the probabilities. In the case of the Markov-switching models the probabilities result directly from the estimation. The Markov-switching model can have two regimes (MS2C) or 3 regimes (MS3C). The MS4C model has two independent chains with two states, one for the mean (column "MS4C means") and one for the variance (column "MS4C variance"). The unconditional probabilities $\bar{\boldsymbol{\pi}}^m$ satisfy $\bar{\boldsymbol{\pi}}^m \boldsymbol{P}^m = \bar{\boldsymbol{\pi}}^m$.

Table C.4: **Time-varying transition probabilities, (multinomial) logit models**

model	LT		PS		MS2L		MS3L					
from	bull	bear	bull	bear	bull	bear	bull		mild bear		strong bear	
to	bull	bull	bull	bull	bull	bull	bull	mild bear	bull	mild bear	bull	mild bear
constant	6.02	−5.31	7.10	−5.69	2.89	−1.58	17.92	15.06	1.11	3.62	−17.44	1.00
inflation	–	–	−1.39	–	–	–	–	0.50	–	–	–	–
			[−0.013]				[−0.021]	[0.021]				
prod. growth	−0.88	–	–	–	–	–	–	–	–	–	–	–
	[−0.007]											
unempl. ch.	–	–	1.00	–	–	–	–	–	–	–	–	–
			[0.009]									
T-bill rate	−1.03	–	−0.35	–	–	–	–	–	–	–	–	–
	[−0.008]		[−0.003]									
term spread	–	–	–	0.67	0.39	–	–	–	–	–	–	–
				[0.011]	[0.020]							
credit spread	–	–	1.57	–	–	–	–	–	–	–	–	–
			[0.014]									
D/P ratio	1.57	0.99	2.72	1.38	–	−0.61	–	–	−0.46	–	–	−1.45
	[0.012]	[0.017]	[0.025]	[0.024]		[−0.078]			[−0.031]	[0.030]		[−0.285]
RM volaility	–	–	–	−0.49	−1.37	–	−1.37	–	–	–	–	–
				[−0.008]	[−0.069]		[−0.059]	[0.059]				
$\bar{\pi}_{sq}$	0.99	0.02	0.99	0.02	0.95	0.15	0.96	0.04	0.07	0.90	< 0.01	0.27

This table shows the estimated coefficients and marginal effects of the predictor variables, when they are linked to the transition probabilities by (multinomial) logit models. The predictor variables have been standardized by subtracting their full-sample mean and dividing by their full-sample standard deviation. In the approaches of LT and PS, we first apply the algorithms to identify bull and bear markets. In the second step we estimate the dynamic logit model in Equation (1). In the MS2L model, the logistic transformation is used to link the predictor variables to the transition probabilities. In the MS3L model the multinomial logistic transformation in Equation (C.1) is used. In that case the coefficients for a switch to the strong bear regime have been set to zero. The variable-transition combinations that subsequently produce the biggest increase in the likelihood function are included in the models. The procedure stops when the remaining combinations of predictor variables and transitions fail to produce an increase in the likelihood function that is significant on the 10%-level. The marginal effects in brackets are calculated for the average forecast probability $\bar{\pi}_{qs}$ reported in the last row of the table. The average forecast probability is calculated as in Equation (C.5). For the two-state approaches, the marginal effect of predictor variable i is calculated as $\bar{\pi}_{qs}(1-\bar{\pi}_{qs})\beta_{qi}$. For the MS3L model, the marginal effect is given by Equation (C.4).

Table C.5: Log likelihood values and information criteria of different model specifications

		LT	PS	MS2	MS2H	MS3	MS4
constant	# parameters	2	2	7	6	14	10
	$\log L$	-158.00	-176.60	-5687.49	-5887.61	-5641.98	-5691.63
	AIC	0.118	0.132	4.198	4.345	4.170	4.203
	BIC	0.122	0.136	4.213	4.358	4.200	4.225
time-varying	# parameters	7	10	10		18	
	$\log L$	-133.97	-137.83	-5666.17		-5626.94	
	AIC	0.104	0.109	4.184		4.161	
	BIC	0.119	0.131	4.206		4.201	
LR		48.06	77.54	42.64		30.09	
df		5	8	3		4	
Pr(0.01)		15.09	20.09	11.34		13.28	

This table shows the log likelihood values of the different models. In the case of the LT and PS methods, we report the log likelihood values of the dynamic logit models in (1). In the case of the Markov-switching models, we report the likelihood of the complete model. The transition probabilities can be constant (corresponding with Table C.3) or time-varying (corresponding with Table C.4). Based on the log likelihood values we calculate the Akaike and Bayesian Information Criterion (AIC and BIC). In the row labeled "LR" we report the likelihood ratio statistic for time-varying versus constant transition probabilities, which has a χ^2 distribution with degrees of freedom listed in the row below. The last row reports 1% critical values of the corresponding χ^2 distribution.

D Robustness Checks

D.1 Robustness of the LT method

Lunde and Timmermann (2004) consider four combinations for the values of the thresholds λ_1 and λ_2 to identify switches between bull and bear markets. They argue that a value of 20% for λ_1 is conventionally used. A lower value of 15% for λ_2 subsequently accounts for the positive drift of the stock market. Other combinations they consider for (λ_1, λ_2) are $(0.20, 0.10)$, $(0.15, 0.15)$ and $(0.15, 0.10)$. Since we conclude that a quick identification of the current state is important when making forecasts, we follow LT and also consider these combinations of lower thresholds.

Lower thresholds lead to a more rapid alternation of bull and bear markets, which as a consequence last briefer. In the identification based on the full sample, the number of cycles, reported in Table D.1, increases from 17 in the original $(0.20, 0.15)$ setting to 23 for the lowest thresholds $(0.15, 0.10)$. Lowering both thresholds has a much stronger effect than lowering only one of them. The average duration decreases for lower thresholds, though the result is less pronounced than in Lunde and Timmermann (2004). This is due the fact that the longest bull market is unaffected by the choice of thresholds. The means of bull and bear markets become more pronounced for lower thresholds, whereas the volatilities do not change much. All changes are smaller than two standard errors of the original estimates.

[Table D.1 about here.]

The comparison in Table D.2 shows that a faster identification of bull and bear markets leads to a better performance. Mean returns rise from 0.97% to 1.37% per week, at the cost of an increase in the volatility from 4.40% to 5.24%. Both the Sharpe ratio and the realized utility increase. An investor would be willing to pay large fees up to 17.8 bps to switch to lower thresholds. Since fees to switch from the standard $(0.20, 0.15)$ are all positive, the fees in Table 2b may underestimate the preference of the standard LT method over the Markov-switching methods. However, these fees correspond with identification, and it is not obvious that working with lower thresholds leads to better forecasts or allocations.

[Table D.2 about here.]

We report the performance of investment strategies that are based on the LT method with different thresholds in Table D.3a. We first consider the strategies without predictor variables. Lower thresholds lead to higher means, which rise from around 0.14% to 0.24% per week, but also to higher volatilities. Part of the increases come from a larger position in the market. Judged by Sharpe ratio and realized utility, the threshold combination $(0.20, 0.10)$ works best, while a lowering of the λ_1 threshold actually leads to lower utility because of the increase in volatility. The fees indicate that the threshold combination $(0.20, 0.10)$ leads to better results than other thresholds. The highest utility in Table D.3a is still lower than the utilities based on the Markov-switching models in Table 3a or a full investment in the market portfolio.

[Table D.3 about here.]

The use of predictor variables does never lead to a better performance. For each threshold combination we see lower means, Sharpe ratios and utilities. Fees are negative and vary from -4.4 bps to -8.0 bps, with p-values varying from 0.08 to 0.01. When we only consider the strategies with predictor variables, the threshold combination (0.20, 0.10) yields the best performance. The differences with the other strategies is significant.

Overall, we conclude that lowering the thresholds improves identification and forecasts. We already concluded that the rules-based methods are better suited for identification. The results for lower thresholds indicate an even larger difference. These results do not carry over to forecasts. The lowest threshold combination leads to the worst performance. The (0.20, 0.10) combination outperformance the (0.20, 0.15) combination, but its performance is still worse than the Markov-switching methods.

D.2 Robustness of the PS method

Pagan and Sossounov (2003) base their algorithm on the algorithm for business cycle identification of Bry and Boschan (1971), and adjust some of the original settings based on the early literature on bull and bear markets, the so-called Dow theory after Charles Dow. They do not investigate how robust their findings are to changes in these settings. We conduct a modest robustness analysis, where we change one parameter at a time. In comparison with LT, PS identifies a few more cycles, so we consider only a relaxation of the constraint on cycle length, which we lower to 52 weeks instead of 70. For the constraint on the length of a phase, we consider values of 12 and 20 weeks. A higher value may lead to less false alarms. The bound on the price change that must be crossed to overrule the phase constraint is currently at 20%. As in the robustness checks for the LT method, we consider a value of 15%. Censoring (currently 13 weeks) will not influence the identification much, but may have considerable impact on forecasts. We consider a lower value of seven weeks and a higher value of 26 weeks, which corresponds with the setting of PS.

The identification is quite robust to these changes. In the basic setting, 17 bull and 17 bear markets result. The results do not change when the phase constraint becomes shorter, the price change bound becomes lower, or when the first and last 7 weeks are censored. A shorter cycle constraint leads to one additional bull and bear market. A longer phase constraint leads to the disappearance of one bull and one one bear market. Longer censoring leads to the disappearance of one bear market. Because results change at most slightly, the further analyses based on identification will also change only slightly. Our conclusion that the PS method works well for identification remains unaffected.

The influence of the parameters on forecasting is larger, though the differences do not point out that relaxing thresholds improves performance. Table D.4 shows that a lower cycle constraint (setting B) leads to a worse performance, in particular a negative average excess return. On the other hand, lowering the phase constraint (setting C) leads to remarkably better performance. The Sharpe ratios are comparable to holding the market portfolio, and utility is considerably higher than for the basic setting. Table D.5 shows that the difference with the basic setting is significant. A higher threshold for the phase constraint does not have much effect on the performance. A lower constraint on the price change does not affect the results at all. Shortening censoring to seven weeks leads to a worse performance, whereas longer censoring does not influence the results much. Since the results do not present a clear pattern, and all realized utilities are below the utilities for the Markov-switching models, we conclude that the preference for the latter is robust to these changes.

[Table D.4 about here.]

[Table D.5 about here.]

Comparing the results with and without predictor variables shows that the inclusion of predictor variables leads to worse performance in every setting. Table D.4 shows that the for using predictor variables is always negative and significant at the 90% level or higher. We conclude again that the use of predictor variables leads to overfitting.

D.3 Risk Aversion

The portfolios and their comparison that we propose in Section 2.3 depend on the coefficient of risk aversion γ. We show here that the choice of γ influences our results, but not the conclusions we draw from them.

To make explicit that w_t^m depends on γ, we write $w_{\gamma,t}^m$ for the optimal portfolio weight in Equation (6). It is proportional to the inverse of γ, $w_{\gamma,t}^m = w_{1,t}^m/\gamma$, with $w_{1,t+1}^m = \mu_{t+1}^m/\psi_{t+1}^m$. Consequently, the average return of an investment strategy is proportional to $1/\gamma$ as well, and the variance is proportional to $1/\gamma^2$. Both effects cancel out in the Sharpe ratio.

The realized utility in excess of the riskless investment, $V(w_{\gamma,t}^m) - V(0)$, is also proportional to the inverse of γ,

$$\begin{aligned}
V(w_{\gamma,t}^m) - V(0) = V\left(w_{1,t}^m/\gamma\right) - r_{t+1}^{\mathrm{f}} &= \frac{1}{\gamma} w_{1,t}^m r_{t+1}^{\mathrm{e}} - \frac{1}{2}\gamma \left(\frac{1}{\gamma} w_{1,t}^m \left(r_{t+1}^{\mathrm{e}} - \mu_{t+1}^m\right)\right)^2 \\
&= \frac{1}{\gamma}\left(w_{1,t}^m r_{t+1}^{\mathrm{e}} - \frac{1}{2}\left(w_{1,t}^m (r_{t+1}^{\mathrm{e}} - \mu_{t+1}^m)\right)^2\right) = \frac{1}{\gamma}\left(V(w_{1,t}^m) - V(0)\right).
\end{aligned}$$

This proportionality caries over to the value of method m in excess of the value of a riskless investment. Because the fee is calculated by the difference between the value of two methods, it is proportional to $1/\gamma$, too.

An increase (reduction) in risk-aversion will lead to less (more) extreme results in Tables 2a and 3a. Means, volatilities, and switching fees will all decrease (increase) by the same relative amount, whereas Sharpe ratios do not change. The ordering of the strategies will not be affected. Because the same effects apply to simulations as well, the standard errors of the fees in Table 2b are proportional to $1/\gamma$, while the p-values in Table 3b are unaffected. So, fees that are significantly different from zero remain significant, independently of the choice of γ. Therefore, our conclusions for the preferences of the different methods do not change.

Table D.1: Full Sample Moments of Bull and Bear Markets for Different Thresholds in the LT method

λ_1		0.20	0.20	0.15	0.15
λ_2		0.15	0.10	0.15	0.10
bull	number	15	17	17	23
	duration	123	101	111	78
	mean	0.38	0.42	0.39	0.46
		(0.04)	(0.04)	(0.04)	(0.04)
	vol.	1.88	1.86	1.90	1.87
		(0.07)	(0.07)	(0.07)	(0.07)
bear	number	15	17	17	23
	duration	58	59	49	40
	mean	-0.66	-0.59	-0.73	-0.74
		(0.09)	(0.08)	(0.09)	(0.09)
	vol.	2.59	2.53	2.57	2.51
		(0.17)	(0.15)	(0.16)	(0.15)

We report the number and the duration, and the mean and volatility of returns for bull and bear markets for different values for the thresholds λ_1 and λ_2 in the LT method. See Table 1 for further explanation.

Table D.2: Performance Measures and Fees Based on Full-sample Identification by the LT method with different thresholds

(a) Performance Measures

λ_1	0.20	0.20	0.15	0.15
λ_2	0.15	0.10	0.15	0.10
av. abs. weight	2.09	2.21	2.18	2.52
mean (in %)	0.97	1.04	1.07	1.37
volatility (in %)	4.40	4.56	4.65	5.24
Sharpe ratio (ann.)	1.58	1.64	1.67	1.88
utility ($\times 10^{-4}$)	55.4	59.0	60.0	73.1

(b) Fees (in bps)

(λ_1, λ_2)	(0.20, 0.10)	(0.15, 0.15)	(0.15, 0.10)
(0.20, 0.15)	3.6	4.6	17.8
	(3.5)	(3.5)	(6.3)
(0.20, 0.10)		1.0	14.2
		(5.1)	(5.2)
(0.15, 0.15)			13.2
			(5.7)

Panel (a) reports performance measures of investment strategies that use the LT method with different thresholds λ_1 and λ_2 for input. Panel (b) reports the fees that an agent would be willing to pay to exchange two methods (row for column). See Table 2 for further information.

Table D.3: Performance Measures and Fees Based on Forecasts by the LT method with different thresholds

(a) Performance Measures

(λ_1, λ_2)	Market	(0.20, 0.15)		(0.20, 0.10)		(0.15, 0.15)		(0.15, 0.10)	
Transition		C	L	C	L	C	L	C	L
av. abs. weight	1	1.76	1.93	1.87	2.00	1.82	2.05	2.13	2.27
mean (in %)	0.10	0.14	0.10	0.24	0.19	0.16	0.13	0.24	0.17
volatility (in %)	2.31	4.36	4.39	4.43	4.38	4.55	4.74	5.20	5.11
Sharpe ratio (ann.)	0.33	0.23	0.16	0.39	0.31	0.26	0.20	0.33	0.23
utility ($\times 10^{-4}$)	4.8	-26.5	-32.1	-18.1	-22.4	-28.9	-36.9	-38.0	-43.3

(b) Fees (in bps)

(λ_1, λ_2)	(0.20, 0.15)		(0.20, 0.10)		(0.15, 0.15)		(0.15, 0.10)	
Transition	C	L	C	L	C	L	C	L
Market	-31.3	-36.9	-22.9	-27.2	-33.7	-41.7	-42.8	-48.1
	[> 0.99]	[> 0.99]	[0.98]	[0.99]	[> 0.99]	[> 0.99]	[> 0.99]	[> 0.99]
(0.20, 0.15, C)		-5.6	8.5	4.1	-2.3	-10.4	-11.5	-16.8
		[0.95]	[0.05]	[0.24]	[0.71]	[0.97]	[0.91]	[0.97]
(0.20, 0.15, L)			14.0	9.7	3.2	-4.8	-5.9	-11.2
			[0.01]	[0.06]	[0.32]	[0.81]	[0.71]	[0.87]
(0.20, 0.10, C)				-4.4	-10.8	-18.8	-19.9	-25.3
				[0.96]	[0.92]	[0.99]	[0.99]	[> 0.99]
(0.20, 0.10, L)					-6.4	-14.5	-15.6	-20.9
					[0.78]	[0.96]	[0.97]	[> 0.99]
(0.15, 0.15, C)						-8.0	-9.1	-14.5
						[0.99]	[0.88]	[0.94]
(0.15, 0.15, L)							-1.1	-6.4
							[0.56]	[0.78]
(0.15, 0.10, C)								-5.3
								[0.92]

See Table 3 for explanation. The different settings in the LT method are indicated by the values for λ_1 and λ_2 and the choice for transition probabilities, which can be constant (C) or time-varying (L).

Table D.4: Performance Measures Based on Forecasts by the PS method for different settings

Setting	Description	Trans.	abs. av. weight	mean (in %)	volatility (in %)	Sharpe ratio	utility ($\times 10^{-4}$)
-	Market	-	1	0.105	2.31	0.33	4.8
A	basic	C	1.61	0.051	3.70	0.10	-22.1
		L	1.86	0.043	4.16	0.07	-33.1
B	shorter cycle, 52 weeks	C	1.93	-0.053	4.42	-0.09	-48.3
		L	2.03	-0.121	4.64	-0.19	-61.4
C	shorter phase, 12 weeks	C	1.04	0.097	2.43	0.29	2.5
		L	1.57	0.168	3.61	0.34	-8.6
D	longer phase, 20 weeks	C	1.61	0.054	3.70	0.10	-21.8
		L	2.00	0.048	4.44	0.08	-38.6
E	smaller change, 15%	C	1.61	0.051	3.70	0.10	-22.1
		L	1.86	0.043	4.16	0.07	-33.1
F	shorter censoring, 7 weeks	C	1.61	0.012	3.69	0.02	-26.1
		L	1.86	0.006	4.15	0.01	-36.8
G	longer censoring, 26 weeks	C	1.61	0.062	3.70	0.12	-21.0
		L	1.86	0.053	4.16	0.09	-32.0

See Table 3 for explanation. The different settings in the PS method are described by their change with repect to the basic setting of $\tau_{\text{window}} = 32$, $\tau_{\text{cycle}} = 70$, $\tau_{\text{phase}} = 16$ and $\zeta = 20\%$.

Table D.5: Fees Based on Forecasts by the PS method for different settings

	A,C	A,L	B,C	B,L	C,C	C,L	D,C	D,L	F,C	F,L	G,C	G,L
Market	-26.9	-37.9	-53.1	-66.1	-2.3	-13.4	-26.6	-43.4	-30.9	-41.6	-25.8	-36.8
	[> 0.99]	[> 0.99]	[> 0.99]	[> 0.99]	[0.60]	[0.93]	[> 0.99]	[> 0.99]	[> 0.99]	[> 0.99]	[> 0.99]	[> 0.99]
A, C		-11.0	-26.2	-39.3	24.6	13.5	0.3	-16.5	-4.0	-14.7	1.1	-9.9
		[0.91]	[> 0.99]	[> 0.99]	[< 0.01]	[0.10]	[0.02]	[0.96]	[0.89]	[0.94]	[0.11]	[0.89]
A, L			-15.2	-28.2	35.6	24.5	11.3	-5.4	7.1	-3.7	12.2	1.1
			[0.88]	[0.99]	[< 0.01]	[0.01]	[0.09]	[0.92]	[0.20]	[0.86]	[0.08]	-
B, C				-13.1	50.8	39.6	26.5	9.7	22.2	11.5	27.3	16.3
				[0.98]	[< 0.01]	[< 0.01]	[< 0.01]	[0.23]	[0.02]	[0.18]	[< 0.01]	[0.10]
B, L					63.9	52.7	39.5	22.8	35.3	24.6	40.4	29.3
					[< 0.01]	[< 0.01]	[< 0.01]	[0.03]	[< 0.01]	[0.02]	[< 0.01]	[0.02]
C, C						-11.1	-24.3	-41.1	-28.6	-39.3	-23.5	-34.5
						[0.93]	[> 0.99]	[0.99]	[> 0.99]	[> 0.99]	[> 0.99]	[> 0.99]
C, L							-13.2	-29.9	-17.4	-28.2	-12.3	-23.4
							[0.90]	[0.98]	[0.94]	[0.99]	[0.89]	[0.98]
D, C								-16.7	-4.2	-15.0	0.9	-10.2
								[0.97]	[0.90]	[0.94]	[0.23]	[0.89]
D, L									12.5	1.8	17.6	6.5
									[0.08]	[0.34]	[0.03]	[0.06]
F, C										-10.7	5.1	-6.0
										[0.90]	[0.08]	[0.77]
F, L											15.8	4.8
											[0.05]	[0.09]
G, C												-11.0
												[0.91]

See Table 3 for explanation. The different settings in the PS method are described by the letters as indicated in Table D.4. Because setting E leads to identical results as setting A, it is omitted from the table.

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